



THE ARCHITECTONIC ENCODING OF THE MINOR LUNAR STANDSTILLS IN THE HORIZON OF THE GIZA PYRAMIDS

Hossam M. K. Aboufotouh

Faculty of Environmental Design, King Abdulaziz University, Jeddah, KSA

Faculty of Fine Arts, Minia University, Minia, Egypt

(fotouh@mail.com & haboufotouh@kau.edu.sa)

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ABSTRACT

The paper is an attempt to show the architectonic method of the ancient Egyptian designers for encoding the horizontal-projections of the moon's declinations during two events of the minor lunar standstills, in the design of the site-plan of the horizon of the Giza pyramids, using the methods of descriptive geometry. It shows that the distance of the eastern side of the second Giza pyramid from the north-south axis of the great pyramid encodes a projection of a lunar declination, when earth's obliquity-angle was $\sim 24.10^\circ$. Besides, it shows that the angle of inclination of the causeway of the second Giza pyramid, of $\sim 13.54^\circ$ south of the cardinal east, encodes the projection of another lunar declination when earth's obliquity-angle reaches $\sim 22.986^\circ$. In addition, it shows the encoded coordinate system in the site-plan of the horizon of the Giza pyramids.

KEYWORDS: Giza Pyramids, Causeway, Archaeoastronomy, Moon, Minor Lunar Standstills, Obliquity.

1. INTRODUCTION

The term standstill was first introduced by Alexander Thom in 1972 (Sim, 2003) and means solstice or slow motion (Judith S. Young, 2006). Similar to associating the term solstice to the sun, the term standstill refers to the slow motion of the moon that occurs every ~ 9.3 years at one of the two extremes of its observed range of declinations or azimuths. Tome *et al* (1975, 19-30) showed that some ancient European cultures recorded it in their megalithic horizons, and discussed the possibility that Stonehenge was a lunar observatory. That is in terms of encoding the azimuths of moonrise and moonset during the year of the lunar standstill. In Egypt, none has found yet any ancient lunar observatory. However, Al-Maqrizi, the medieval historian (1364-1442AD), mentioned narratives about the astronomical daily records in ancient Egypt, citing early Coptic historians. He wrote "the priest who spent seven years in studying any of the seven moving orbs: Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn, was called *Baher*; and the one who studied all of them was called *Kater*, implying the master priest¹ (Al-Maqrizi, 1846). Recent researches support the narratives of Al-Maqrizi. Aboufotouh (2007) showed that Dendera Zodiac² records the declination of the full moon that appears at $\sim 13.6^\circ$ west of the meridian and $\sim 30^\circ$ south of the zenith; it records the full moon as was observed from the latitude of 30° north, during the vernal equinox. In Napta in Upper Egypt, Malville *et al* (1998) had found a primitive observatory site, and concluded that it dates back to ca 4800 BC and encodes the sightline of summer solstice. Aboufotouh (2002) showed that the design-locations of the three Giza pyramids were set based on recording the observed motion of the sun when earth's obliquity angle O_t was $\sim 24.10^\circ$, which denotes that the encoded date in the design of Giza Pyramids' horizon³ is $\sim 3055-3065$ BC. Magli (2013) showed that if one stands at the eastern end of the causeway of the second Giza pyramid at the time of the setting sun

in summer solstice, one will observe its alignment as it coincide with the mid point of the sunset angle from due west⁴. Besides, the tilts of the entrance passages of the Giza Pyramids were set based on using relativistic mathematical equations. The equations link the latitude of the pyramid λ with the obliquity of time $O_t = \sim 24.10^\circ$ and the two extreme values of earth's obliquity rang as a frame of reference: 24.30° and 21.672° , and/or its median value 22.986° , as was thought by the pyramids' designer in his days (Aboufotouh, 2007).

This paper is an endeavor to show that the ancient Egyptian designer also used a method of descriptive geometry for encoding the horizontal projection of the lunar declinations in the site plan of Giza Pyramids plateau. It shows the encoded horizontal projection of moon's declination during the minor lunar standstill when O_t was $\sim 24.10^\circ$, which identifies the distance of the eastern side of the second Giza pyramid from the cardinal north-south axis of the great pyramid. Besides, it shows the encoding of another lunar declination in the alignment of the causeway of the second Giza pyramid, during other event of minor lunar standstill when earth's O_t reaches its mean of $\sim 22.986^\circ$, according to the opinion of the ancient designer/astronomer.

2. THE ASTRONOMICAL DESIGN OF THE HORIZON OF GIZA PYRAMIDS

Fig.1a shows a north-south cross-section in a spherical coordinate system, for the latitude λ of 30° north, and $O_t = 24.30^\circ$, and where the observer looks towards the west. Its center is imaginary the center of the earth, i.e., it is a geocentric system, while both observations and design outputs are topocentric. The lines P_1-P_3 and P_2-P_4 are the projections of the two planes of the daily motion of the sun during summer and winter solstices, respectively. The designer of the Great Pyramid in Giza used similar cross-section for aligning its four internal shafts, using O_t equals $\sim 24.10^\circ$ and $\sim 21.672^\circ$ (Aboufotouh 2005), see : ig.1b. If

one rotated the plane of that north-south cross-section (Fig.1a) clockwise for 30°, and afterwards rotated it around the axis Y_n-Y_s for 90°, towards the west, it will become a horizontal plane, i.e., a circular horizon

where its contents of lines and curves are imaginary projected from that plane on its ground site⁵, as shown in Fig. 2.

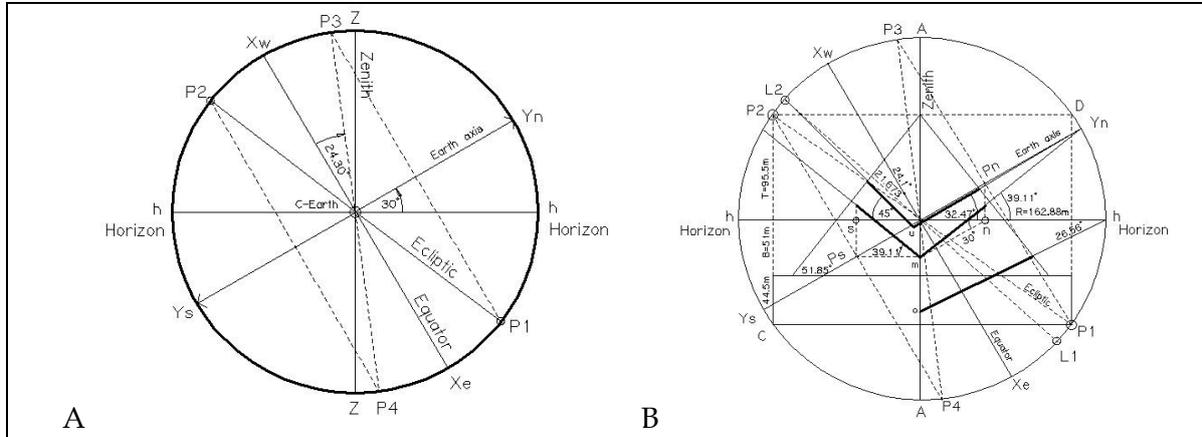


Figure 1. Using the cross section in a spherical coordinate system in pyramids design: (A) shows the north-south cross-section in a spherical coordinate system at the latitude of 30° north, for the obliquity angle 24.30°; and (B) shows the astronomical design concept of the Great Pyramid in Giza, based on a design radius equivalent to 168.88m, which it is the radius of a circle enclosing its square base; while conserving two obliquity angles: 24.10° denoted by the limits of the projection-lines P_1-P_3 and P_2-P_4 ; and 21.672° denoted by the line L_1-L_2 , and the position of the north pole at Y_n , and its southern mirror, that upon which the directions and ends of its internal shafts were designed (Aboulfotouh, 2005).

Accordingly, the line Y_n-Y_s that represents the earth's spin axis, in the cross section, will become the geographic north-south axis of that circular-horizon. No matter where the horizon model will be implemented any where on earth, its center c imaginary represents the center of the earth. Besides, its cardinal east-west axis X_e-X_w and the diameter P_1-P_2 , will imaginary represent the horizontal projections of the earth's equatorial plane and the plane of the ecliptic in the horizon's ground-plane, respectively. The paper shows hereafter that the ancient Egyptian designer used this cross-sectional horizon-model in the site plan of Giza Pyramids plateau.

Aboulfotouh (2002) showed that the center of the horizon-model of Giza Pyramids plateau is the point of intersection between the cardinal east-west axis of the Sphinx and the cardinal north-south axis of the Great Pyramid, as shown in Fig. 3, where the horizon's design-radius R was found = ~746m. That value of R complies with the results of Petrie's survey data (Petrie, 1882, p125), and gives correct and meaningful

astronomical positions for the three pyramids, with regard to the observed daily motion of the sun, taking into consideration the value of the implementation tolerances in megalithic construction-survey, i.e., ~1/1000.

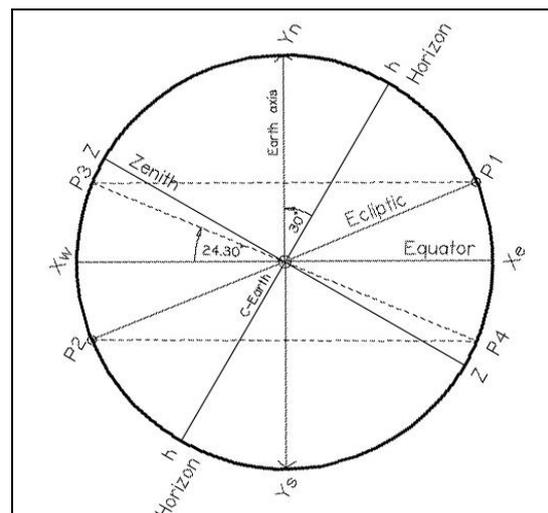


Figure 2. The generated horizon-plan after rotating the spherical cross-section, in fig.1a, 30° clockwise, and afterwards rotating it 90° anticlockwise around Y_n-Y_s . At the end, $Y_n, Y_s, X_e,$ and X_w will denote the cardinal directions: north, south, east, and west, respectively.

3. THE LUNAR DATA AND THE METHOD FOR RECKONING THE DECLINATIONS OF THE MOON

The current earth's obliquity to its orbits O_t is about 23.45° (Williams, 2006); and the mean geocentric inclination of the moon's orbit to ecliptic $i_g=5.15^\circ$, which is the mean between two values⁸: 4.95° and 5.33° (Malville, *et al* 1993).

Hence, the geocentric declination of the moon Δ fluctuates between two values: the minimum $\Delta_m = (O_t - i_g)$, and the maximum $\Delta_x = (O_t + i_g)$. They are the geocentric declinations of the minor and major lunar standstills, where their current approximate values are: 18.28° and 28.58° , respectively (Williams, 2006).

These values are the inclinations of the full moon, from the earth's equatorial plane, when the moon meets the meridian of the place of observation. Similarly, for $O_t=24.10^\circ$, $\Delta_m=18.95^\circ$ and $\Delta_x=29.25^\circ$, approximately.

Besides, the moon spends about 9.3 years to swing between the two declinations: Δ_{xg} and Δ_{mg} , i.e., it spends about 18.61 years between two succeeding minor standstills⁹. The last minor lunar standstill was in February 1997, and the next one will be in October 2015 (Vincent, 2005).

One cannot observe the true value of the geocentric declination of the moon while standing on earth. It is due to the effect of both the parallax and the atmospheric refraction¹⁰ that should be taken into consideration. Their values should be added to i_g , where the result is the moon's topocentric declination, when observed from the earth's surface.

For reckoning the parallax, Fig.4 shows an observer at c_1 standing at $\lambda = 29.974^\circ$ north ($\sim 30^\circ$), during the minor lunar standstill.

The angle φ is the maximum geocentric declination of the full moon from the zenith of the place and the angle θ is its associated topocentric observed value. Hence, the value of θ could be reckoned using the equation¹¹: $\text{Atan } \theta = (R_1 * \sin \varphi) / ((R_1 * \cos \varphi) - R_2)$.

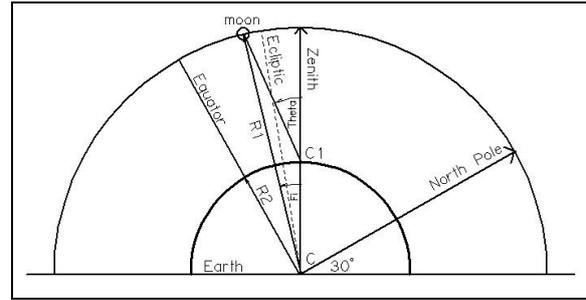


Figure 4. Half cross-section in the semi-elliptical sphere of the moon's orbit around the earth. It is perpendicular to the plane of the moon's orbit and passes through the line of apsides that part of it is being represented in the figure by the line R_1 , which it is the shortest radius of the moon's orbit. The point C is the center of the earth, and the point C_1 is the place of an observer at the latitude $\sim 30^\circ$ north of the equator, and R_2 is the mean radius of the earth. The figure is not to scale.

Where R_1 is the mean radius of the moon's orbit, R_2 is the volumetric mean radius of the earth; they are 384,469 km and 6,371 km respectively (Williams, 2006); and φ equals $(\lambda - O_t + i_g)$, where λ is the latitude of the place of observation. For $O_t = 24.10^\circ$ and $\lambda = 29.974^\circ$ north, the difference between φ and θ is $\sim 0.184^\circ$, which should be added to i_g of 5.15° , and the result would be the observed topocentric inclination i_{mp} (or i_{xp}) of the moon's orbit to the plane of the ecliptic. The effect of atmospheric refraction can be neglected at the elevations close to the zenith¹².

Accordingly, if $\lambda=29.974^\circ$ north, and $O_t = 24.10^\circ$, $i_{mp} = \sim 5.334^\circ$ for the minor lunar standstill¹³. Similarly, for $O_t=22.968^\circ$, the corresponding i_{mp} is $\sim 5.352^\circ$. In order for an ancient designer to encode the declination of the full moon during the minor lunar standstill for the obliquity angles: 24.10° and 22.986° , he might encode the corresponding Δ_{mp} as: $\sim 18.766^\circ$ and $\sim 17.634^\circ$ respectively in the design of his megalithic horizon, where $\Delta_{mp} = O_t - i_{mp}$.

However, if the first inclination was an observed value in his days, i.e., it is not derived from a reckoning process, he might assume that $i_{mp}=5.334^\circ$ for any obliquity angle, and his supposition for the encoded declination Δ_{mp} of the moon might be hence equal $\sim 17.652^\circ$ for $O_t=22.986^\circ$.

4. THE ARCHITECTONIC MAPPING OF THE MINOR LUNAR STANDSTILL.

Fig. 5 shows photos of the Giza pyramids plateau; and the causeway of the second Giza pyramid. Based on Google Earth data (of 7/12/2013) the ground length of causeway, from the western entrance of the valley temple to the eastern entrance of mortuary temple is ~496m; of which only ~90m from the valley temple remain with parapets¹⁴; and the orientation of its axis is almost 13.5° south of the cardinal east.

In 2006, Nell *et al* (2012, p19) surveyed the boundaries of the causeway, and concluded that the orientation of the north and south boundaries of the causeway is 13° 26' and 13° 33' south of the cardinal east, respectively.

It confirms the Google Earth data. The causeway intersects with the east-west axis of the horizon and the Sphinx at point X where its distance¹⁵ from the north south axis of the Great pyramid is ~193m, as shown in : ig-5a, and : lg-6.

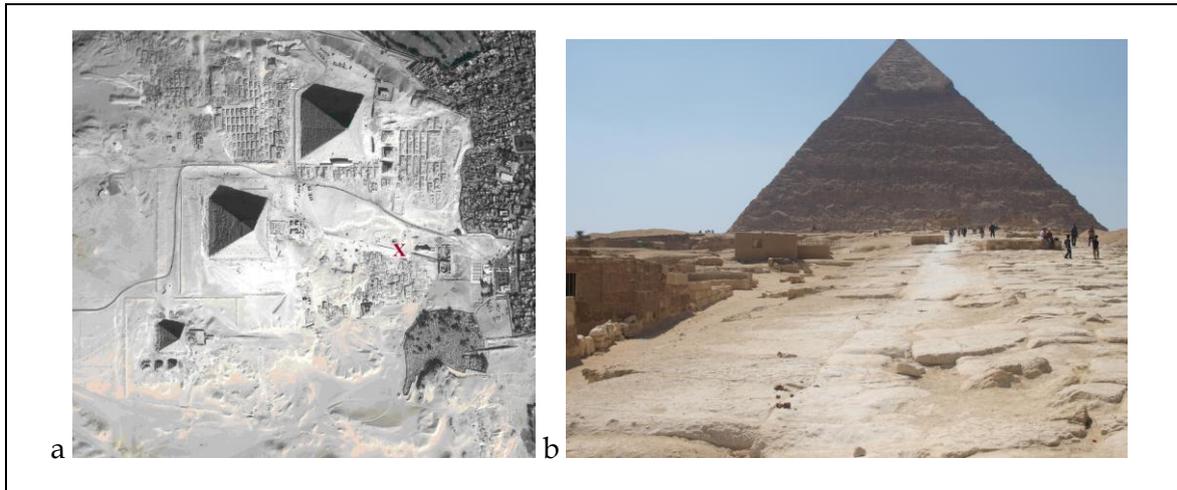


Figure 5. The aerial and ground photos of the Giza pyramids: (A) a Google earth photo of the Giza Pyramids plateau, where X in red marks the point of intersection between the axis of the causeway of the second pyramid and the east-west axis of the Sphinx; and (B) is a photo taken at X and looking towards the west, where the causeway is inclined to the right side from due west by almost 13.54°.

Fig.6 shows the circular horizon of the Giza Pyramids. It is similar to the rotated cross section in Fig.2. The points: M_1 and M_2 represent the observed positions of the full moon, when it crosses the meridian of the place, during two events of minor lunar standstill: M_1 for $O_t = \sim 24.10^\circ$; and M_2 when O_t reaches $\sim 22.986^\circ$. The line $C-M_1$ that corresponds to $O_t = \sim 24.10^\circ$, intersects with the east-west axis of the second pyramid at point Y, which it is also the center of the eastern side of the base of the second pyramid. That intersection identifies the east-west position of the second Giza pyramid, implying the encoding of the declination of the moon during the minor lunar stand still for $O_t = \sim 24.10^\circ$, in the east-west position of the second Giza pyramid. The side-dimensions¹⁶ of the right-angled triangle

YCT confirm the projected lunar declination angle $\Delta_{mp} = 18.766^\circ$; where $\tan 18.766^\circ = YT/TC = \sim 77.05/226.78$. Since Fig.6 is a cross-section and a horizon-plan in the same time, the projection of the points M_2 on the line $h-h$ is the point V_2 . From V_2 , if we draw a line towards the east, and parallel to the cardinal axis $E-W$ in the horizontal-plan (or the equator in the cross-section) it will meet the circular circumference of the horizon at point L_2 . Now, the triangle $M_2-V_2-L_2$ corresponds to the event of the minor lunar stand still at the time when $O_t = \sim 22.986^\circ$. By reckoning, if $\Delta_{mp} = O_t - i_{mp} =$ angle M_2-C-W , and β is the angle M_2-X-W , which it is the inclination of the axis of the causeway M_2-L_2 on the east-west cardinal direction; then, $\tan \beta = R \sin \Delta_{mp} / (R \cos \Delta_{mp} + CX)$. That is, if $R = 746m$, $CX = 193m$,

and $\Delta_{mp} = 17.65143^\circ$, then $\beta = \sim 13.544^\circ$. The value of β conforms to the survey result of Nell *et al* (2012, p19) of $13^\circ 33'$. It proves the justification of the astronomical purpose of the alignment of the causeway of the second Giza pyramid as to encode the declination of the moon during the minor lunar standstill when earth's obliquity angle reaches its mean value of 22.986° as was

thought by the ancient Egyptian designer. Concerning the encoded years that the two lunar declinations mark, counting from 2016AD, and using the interval of 18.61 years, in the past the year 3065BCE was a year of minor lunar standstill and meets $O_t = \sim 24.10^\circ$, and in the future, 5478AD will be a year of minor lunar standstill and meets $O_t = \sim 22.986^\circ$.

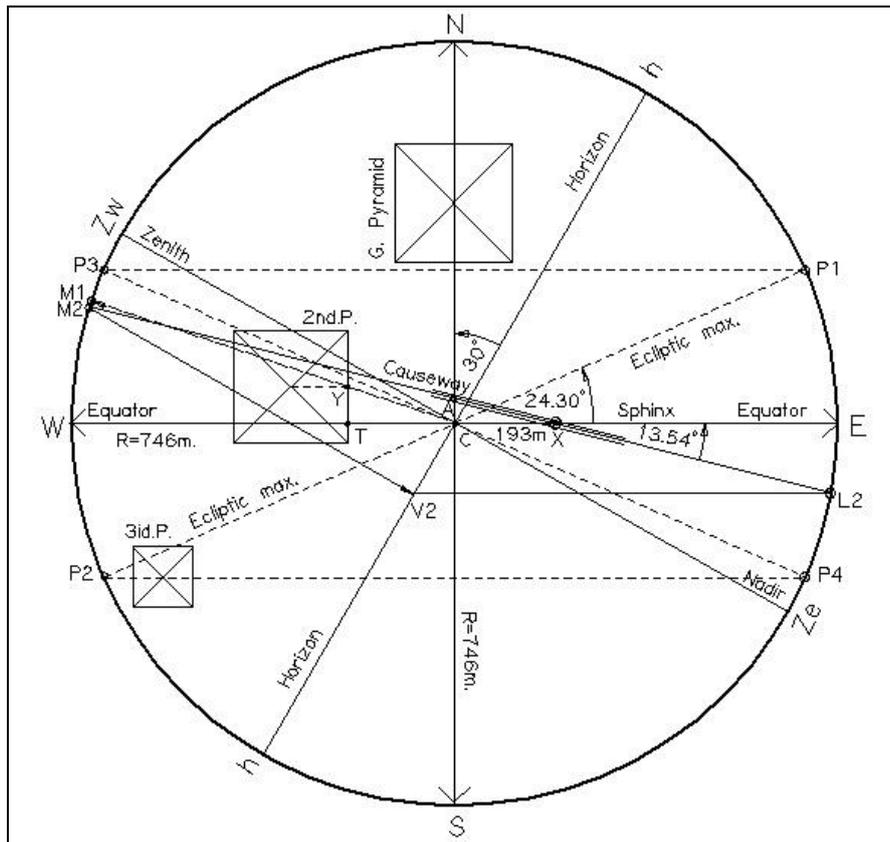


Figure 6. The circular horizon of the Giza pyramids. It is a horizon plane and a vertical cross section in the same time, and its center C is imaginary the center of the earth. Hence, the point Z_w represents the zenith of the place, the north south coordinate-line N-S represents the spin axis of the earth, and the east-west coordinate-line E-W represents the projection of the plane of the earth's equator. The line P_1-P_2 represents the projection of the plane of the ecliptic with its assumed maximum inclination of 24.30° to the earth's equatorial plane¹⁷.

4. CONCLUSION

The paper showed the architectonic method of the ancient Egyptian designers for encoding the horizontal-projections of the moon's declinations during two events of the minor lunar standstills, in the design of the site-plan of the horizon of the Giza pyramids, using the methods of descriptive geometry.

Its findings are consistent with the results of previous papers on the architecton-

ic and astronomical design of the Giza Pyramids and their horizon. It implies the ancient Egyptian designers were encoding astronomical information using the geometrical language. Using descriptive geometry and algorithms, the information they encoded were not only on the daily motion of the sun and the earth obliquity range but also on the motion of the moon, particularly encoding the moon's declinations during specific events of the minor lunar standstill.

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NOTES

¹ Al-Maqrizi (1846) also said that every day in the hall of the King, the *Kater* asks each of the *Bahers*: what is the position of his friend (his orb) today, where upon he informs the king about what should be done in that day concerning various activities, e.g., manufacture, agriculture, etc.

² Its design is the spread out of the sky dome, where its radius is 90°; and the position of its tropical and sidereal zodiacs denote the date 3430BCE approximately (Aboulfotouh, 2007).

³ The encoded date does not imply the date of construction of the Giza Pyramids, but denote "an important point in the frame of time" from the point of view of the designer. In this regard, Ramsey *et al* (2010, p1556) concluded that the dates of the reign of Djoser and Sneferu are 2691 and 2649 BCE respectively. Ramsey's findings conform partly to the story mentioned by Al-Maqrizi (1849) who said, "Pyramids construction had started 400 years before the date of the cataclysmic event that, based on the papyrus of king So-ra-eid, will occur when the star Regulus enters the first minute of Cancer, i.e. in 2350 BCE (Aboulfotouh, 2007).

⁴ Magli (2013) based his opinion on that the causeway is oriented 14° north of the west, and the angle of the setting sun on the horizon in summer solstice, at the latitude of Giza, is ~ 28° north of the west.

⁵ The position of the eye of the beholder is also been rotated anticlockwise for 90°, from the east to the zenith of the horizon.

⁶ Its crest and body is the zenith and the vertical axis of an imaginary sky dome that its radius is the same as that of the horizon R that equals =746m.

⁷ The encoded value of $a=28.13^\circ$, that conforms to the value of $R=746m$ and to Petrie's survey data of site-measurements (Petrie, 1882), matches the output of using the geocentric equation: $\sin a = (\sin O_t / \cos \lambda)$, for the obliquity angle $O_t=24.10^\circ$, and the latitude $\lambda=30^\circ$ (Aboulfotouh, 2002). Taking into consideration the implementation tolerance, based on Google Earth data, the true λ of the center of the horizon of Giza pyramids is ~29.974°; but the value of the ancient observation and design seem was 30°. Besides, the value of a follows the ancient Egyptian binary subdivision of the circle (Aboulfotouh, 2012), i.e., 28.13° equals ~ (1/16 + 1/64) of the circumference of a circle, it also raises the question: why the parallaxes and the atmospheric refraction are not included in it?

⁸ The orbit of the moon is tilted with respect to the earth's orbit by 5°9'; perturbations cause it to vary from 4°57' to 5°20' (Malville, *et al* 1993, p18), and Williams (2006) mentioned $i_g=5.145^\circ$, but his figure for the declination is reckoned based on $i_g=5.15^\circ$ similar to the value of Malville.

⁹ Due to the gravitational affect of the sun, the moon's orbit processes with a period of 18.61 years. The major standstill is followed by the minor standstill 9.3 years later (Malville *et al*, 1993, p19).

¹⁰ The lunar parallax is the difference between the position of the moon in the sky, as viewed (theoretically) from the center of the earth, and (actually) from somewhere on the earth's surface; and the atmospheric refraction is the bending of light downwards as it passes through the atmosphere, which causes astronomical bodies to appear higher in the sky than they are in reality (Ruggles, 2005, p.234 & p.364).

¹¹ A revised equation based on the explanation of Pederson (2010, pp203-214) concerning Ptolemy's approach on reckoning the Parallax [V, 17; her 1, 428]; using different notations, Pederson used the equation: $\sin(\theta - \varphi) = R_2 \sin \theta / (R_1 - (R_2 \cos \theta))$.

¹² Tycho Brahe assumed that the atmospheric refraction is 0.0 above 45° altitude/elevation (Leverington, 2003).

¹³ Equation-1 would give i_{sp} equal 5.131° for the same λ and O_t during the major lunar standstill.

¹⁴ Based on site measurements, the width of each parapet was 2.30m enclosing narrow passage of 2.0m, and the construction width of the bedrock of the causeway is 19.60m.

¹⁵ Based on the Google Earth data (2013), and the aerial survey maps of Cairo Engineering & Manufacturing Co. (1996).

¹⁶ The east-west distance from center of the Great pyramid to center of the Second pyramid is 13165.8 inches; and the mean length of the side of the second pyramid's base is 8474.9 inches (Petrie, 1883, p96 & p125), where one inch = 0.0254 meter, i.e., $TC=226.78m$; and based on (Aboufotouh, 2002) the design value of $YT=77.05m.$, as shown in Fig.3.

¹⁷ If one rotated Fig.6 clockwise until the line $h-h$ becomes horizontal, i.e., the zenith up, one will observe it as a cross-section in a coordinate system, and the three Giza Pyramids will appear as three celestial bodies before their dark fall.