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CRITICAL ASSESSMENT OF THE BARREL VAULT GEOMETRY AND STRUCTURE OF THE OLDEST MACEDONIAN TOMB OF EURYDICE IN VERGINA

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ABSTRACT

This article is concerned with the particular case of the semi-cylindrical barrel vaulted Macedonian tomb of "Eurydice", the earliest known Macedonian contribution to the development of vaulted Hellenistic architecture and structure. This tomb is considered as a critical case in the problematic chronology of barrel vault application in early Hellenistic architecture (around 340 BCE), represented by the Macedonian tombs. Still, perhaps the most striking features of the tomb– the physical structures of the barrel vault – have not been subjected to geometrical and engineering structural analysis. The paper's scope focuses on the structural assessment of Eurydice tomb construction. The paper primary objectives are to develop an understanding of the structural mechanics of Eurydice barrel vault within the supporting bearing walls, and to investigate the tomb unique geometrical and structural design. In order to accomplish this task, a general historical and structural behavior study was carried out regarding the barrel vault and its geometry, by following the theories developed by recent researchers, in order to compare these results and to rationalize them in this unique example of Eurydice tomb. Mainly, to undertake a task, that till now, was never accomplished. The results of this work show that Eurydice tomb exhibits such a high structural integrity and strength, demonstrating the ability of early Hellenistic architects to construct extremely safe barrel vaulting structures. The paper also argues that, the tomb architect was familiar with the main theoretical laws of structural force mechanics of the barrel vault, which were commonly accredited to the Romans, and that were later on revived until the 18th century.

KEYWORDS: Hellenistic Macedonian architecture; Semi-cylindrical barrel vault; Chronology; Structural techniques; Geometrical and Structural analysis; Computing geometry.

1. GENERAL INTRODUCTION TO THE MACEDONIAN TOMBS ARCHITECTURE

The history of architecture is inextricably linked with the history of construction techniques. Semicylindrical barrel vaulted roofs masonry represent the typical principal structural feature of Hellenistic Macedonian tombs architecture. The Macedonian tombs are a particular category of underground chambered structures, found basically in Macedonia. A circular mound nearly covers these tombs, while a built a passageway "dromos", leads to some of them. These buildings could have sustained huge pressure of earth covering in some cases really enormous tumuli (Andronikos 1987, 12). In Fact, the masonry barrel vaults of the Macedonian tombs are valuable structures for these funeral monumental buildings.

As a structural feature, the principal function of these stone barrel vaults is to cover an area and to carry the load located atop it. They are the sole surviving construction, not only of the period, but also of all ancient Greek architecture. They provide us with priceless material and evidence not only for the development of forms and construction techniques, but also for the means of processing complex features such as plaster, color, wood and marble construction. They appear to sum up perfectly the spirit of Hellenistic architecture (Haddad, 1995, 266; 1999,162; 2012). In fact, they are of significant contribution to the history of Greek architecture.

These tombs were usually constructed of local porous limestone, and were coated with stucco, which in several cases bore painted decoration. They consist of a spacious burial chamber, square or rectangular. However, as shown in Figure 1, they often have an antechamber connecting with the main burial chamber by means of a monumental doorway. The entrance is usually a rich marble door way framed with a door post and lintel, with also marble shutters carved as imitation of the original one of wood, as had been discovered in many of these early Macedonian tombs

In conclusion, through the architecture of the Macedonian tombs, we can follow the beginning of the spread application of the barrel vaulted construction and the architectural treatments of the problem of the connection of functional vault (Miller 1982, 154-155; Haddad, 1999, 163) with the symbolic basic façade, which led to a purely stylistic formed façade with architectural stucco decoration. Furthermore, we can observe and examine the development of the use of the engaged orders, false doorways and windows. However, the extensive use of engaged order implemented in their façades, especially in the early tombs, mainly for decorative and aesthetical purposes, was actually the beginning of separation and disconnection independent from function (Haddad,1999, 162). The result was the creation of a new morphological expression/direction in the architectural spirit, that fully adopted by the early Hellenistic architects (Haddad, 2012).

2. MACEDONIAN TOMBS BARREL VAULT CHRONOLOGICAL DEBATE ALONG WITH THE ARCHITECTURAL AND STRUCTURAL ASSESSMENT OF EURYDICE TOMB

It has been a matter of some debate on whether the Macedonian tombs barrel vault origin lay in Macedonia itself or whether it was imported by those who accompanied Alexander the Great (Miller 1982, 167, 1993, 101; Whitley 2001, 408–210; Chilidis 2008; D'Angelo 2010; Popovic', 2011,166). Boyd (1978, 88-89) in his work "The Arch and the Vault in Greek Architecture", prior to and unrelated to the Vergina/ *Aegae* - the old Macedonian capital- tombs, argued that, due to the lack of archaeological evidence for a period of experimentation with roof types in Macedonia, the barrel-vaulted tomb type was not introduced in Greece and Macedonia until the conquests of Alexander the Great, and was directly copied from the East, when it appears suddenly and fully-developed. Lehmann (1980, 529) agreed with Boyd and assumed that all known barrel vaults in Macedonia are fully developed.

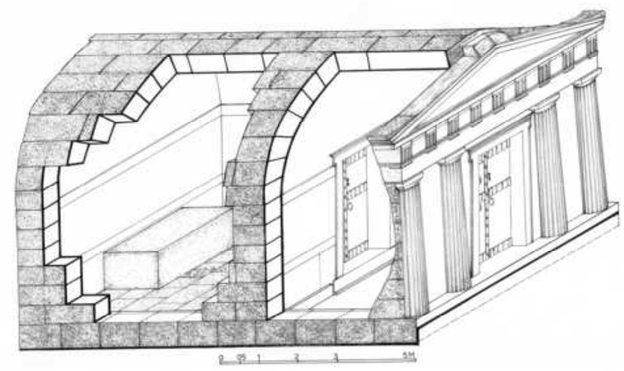

Figure 1. Axonometric of the Macedonian tomb D at Pella. (Haddad, 1999, 165).

It is clear that during the late Classical and early Hellenistic age the barrel vaulted tombs became most spread in Macedonia, just as the beehive *tholos* tombs became widespread in Thrace. However, the discovery of the Royal necropolis tombs at Vergina/ *Aegae*, has led to discussion of problems posed by them; problems of chronology, and the difficulties of deducing date purely from consideration of their architectural features is still open. Never less, there is still discussion going on whether the barrel vault is of local origin and was developed in Macedonia itself, though it should be dated in the time before Alexander conquest (Andronikos, 1987; Haddad,1995,73; Green 1998, 162; Whitley, 2001, 408–210; Chilidis 2008; Hatzopoulos, 2008; D'Angelo, 2010; Errington,2010,Popovic',2011,166, Lane Fox,2011), or it was brought in Macedonia as a mature form and copied from the Near East (Tomlinson, 1977,474,1987; Lehmann, 1982, 437-42**;** Fedak, 1990, 74–76; Faklaris, 1994, 616; Borza and Palagia, 2007; Gill, 2008, 162). The group of this scrutiny's, assume that the Macedonians first had contact with the East at the time of Alexander and not earlier. Though, the issue is mainly related to the earliest documented use of the semi-cylindrical barrel vault roofing structure in Macedonia.

More specifically, Eurydice tomb[1] (dated around 340 BCE) (shown in Fig. 2), and the association of Tomb II with Philip II(336 BCE) in Vergina (as shown in Fig. 3), initiated a debate concerning the use of barrel-vaults in Macedonian tombs. The accepted theory at the time held that, since no Macedonian tomb was dated prior to the last quarter of the 4th century BCE, and therefore Tomb II of Philip II, should be dated to a later period, and specifically to Philip III Arrhidaios (316 BCE) (Borza and Palagia, 2007). This while barrel vault tombs were built in Anatolia before Alexander's conquest, for example, the tomb at Labraunda in Caria, dated to the 340s under Hekatomnid rule (Fedak, 1990, 74–76). Barrel vaults also as architectural features in fortifications and public buildings are not very frequent in the Greek cities (Tomlinson, 1987, 308).

However, the question of the barrel vaulted tombs was comprehensively dealt with by Andronikos (1987, 1-16) and his arguments in favor of the local Macedonian origin of this technique were deemed convincing by Tomlinson (1987, 305-312), who had previously shared Pyllis Lehmann's[2] view on their introduction from the East as a result of Alexander's expedition (Hatzopoulos, 2008, 97). Andronikos advocated the thesis that free arch appeared in Macedonia in the middle of the 4th century BCE (Andronikos, 1987, 12), and barrel vault was invented and improved in the mid 4th century by the builders of large Macedonian tombs.

On the other hand, this hypothesis and argument that the barrel vault copied from the Near East was weak yet when it was first used. Plato refers to a vault (*ψαλιδα*) in his Laws where he describes the ideal burial for a State appointed high priest (Laws XII,947D).

Figure 2. The back interior wall with the false façade of the main chamber of the Macedonian tomb of Eurydice (340 BCE), in Vergina (Andronicos, 1993. p.101, Fig. 57).

In addition, according to Hatzopoulos (2008, 107), "it a priori excluded all possibility of transmission of techniques between the Near East and Greece in the preceding period; because it ignored the fact that stone vaults, as opposed to brick vaults, were first attested in Greece; and because it arbitrarily rejected literary evidence adduced by Fredricksmeyer, Calder, Tomlinson and Andronikos himself that the vault had been known in Greece before the Hellenistic Age".

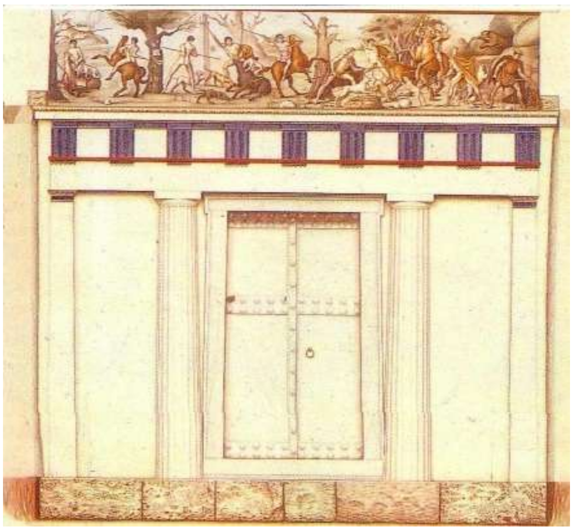

Figure 3. Façade of the Macedonian tomb of Philip II (336BCE) in Vergina with the famous painting of the Hunt on the frieze (Andronicos, 1993. p.101, Fig. 57).

In fact, Macedonia was a vassal state of Persia in the fifth century (Fredricksmeyer, 1981, 333) and Greece had contact with the East as a result of the Persian Wars of the same century (D'Angelo, 2010, 59). Though, after however an intense dispute, fresh archaeological evidence also shows that the theory the copied from east was false. The recent discovery of the barrel vault (5.30 x 9.20 m and 6m high) in the water supply system at Kr'evica is an indication of the local origin of the barrel vault. According to Kr'evica excavator, the results on the basis of gathered evidence suggest that this barrel vault building should be dated not later than the middle of the 4th century BCE (Popovic', 2011,166).

It appears that the Macedonian tombs were the outcome of a lengthy process of evolution which began with the traditional cist graves in which all decoration was executed on the interior walls. In fact, through the enlargement of cist tombs to meet the particular needs posed the larger tombs of the fourth century BCE. According to Andronikos (1987, 10-11),the two 4th century BCE cist tombs at Palatitsia near Vergina show a variation in their roofing approach with the addition of a cross wall or pillars that partition the tomb into two chambers, and also combat the problem of collapsing roofs. In addition, the recent discovery (2012) in Vergina by Kottaridi (as shown in Fig.4**)**, also enhance this theory, in which the final evolutionary stage of the Macedonian tomb involved the transition from horizontal slabs to barrel-vaulted roofs.

Figure 4. The recent discovery (2012) in Vergina by Kottaridi cist grave.

(http://contentmcdn.ethnos.gr/filesystem/images/20140316/low/assets_LARGE_t_420_54322672.JPG)

This large cist tombs of Aegae / Vergina of the 5th century BCE with interior engaged order, are the first instances of burial structures which present problems as to their roofing because of their larger than normal dimensions (as shown in Fig.4**)**. There follows the tomb of Katerini in Pieria, dated from the second quarter of the fourth century BCE, which combines the double chamber with carved connecting door, that is to say of a 'Macedonian tomb' of particularly extensive dimensions with horizontal large flat-roofed ceiling of a cist grave (Haddad, 1995,73-4 Hatzopoulos, 2008, 107).

Actualy, these large cist tombs of the 5th and 4th century BCE attest the tendency in the ruling classes of Macedonia for monumental burial constructions. Nevertheless, not all Macedonian tombs feature this vaulted roof type over the entire structure, but there are some antechambers exhibiting flat roofs.

The rapidly in fact development process, in which new barrel vaulting structures only slightly differed from the original one, was a usual way of development of structures and methods of construction. Indeed, after some experiments to find the safest manner of roofing these large spaces, architects and builders arrived at the solution of the barrel vaulted roof.

Accordingly, when Chilidis examined the ways we structure knowledge in archaeology from hypothesis to theory that can develop to consensus, and how later consensus exercises a conservative influence on the production of new knowledge, he clarified that new evidence that contradicts consensual theories is approached with stronger hostility, and is confronted with higher demands of confirmation. Chilidis suggested that the same amount of scrutiny should be applied to the established theories, which are not unchangeable representations of reality, but conventionally shared property of archaeologists (Chilidis,2008).

On the other hand, meanwhile the Macedonian tomb barrel vault was subjected to a long debate regarding to its chronology, there is no study that discus and debate the geometrical and structural behavior of the earliest one , tomb of Eurydice (as shown in Figs.2,5), and the way it keeps its equilibrium and transmits its thrusts. This tomb is completely encased in a protective rectangular construction block of double walls and the vault is also boxed into a rectangular cist-shaped structure, and with 'Π' shaped iron nails in intervals over the entire surface of the vault (Hatzopoulos, 1994, 156, 2008,107).

Actually, the semi-cylindrical barrel vaulted roof of Eurydice tomb, is the only one of the whole Macedonian tombs encased in a rectangular construction, with two rows of blocks above the vault crown area over the key stones part, and lacks a decorated exterior facade. The tomb exterior is plain in the proper cist tomb manner.

The internal back wall of the chamber (shown in Fig. 2) is decorated in the cist tomb approach; the cist is partitioned into the interior decorated with elaborate Ionic engaged order façade.

Haddad (1995,73-74, 1999,162) affirmed that, one can clearly see the first attempts of early Hellenistic architects in the construction of the vault, but also fear and insecurity in the use of the semi-cylindrical vault (as shown in Fig. 5).

Drougou and Saatsoglou-Paliadeli (2004, 60) suggest that such a construction indicate a developmental phase and uncertainty on the part of the builders about the stability of the tomb, meanwhile Hatzopoulos (2008,107) affirmed that this composition left no doubt that, the architecture of the Macedonian tomb was the result of a local evolution responding to the need to provide sufficiently resistant roofing system for large underground structures, and thus was not borrowed from the East.

The artistic final stage of this evolution can be seen in the Tomb façade of Philip II, where in less than five years after Eurydice tomb, the engaged order facade tomb became the prime characteristic of the Macedonian tombs. Just as an example of this rabidly evolution, we are mentioning one tomb in the vicinity of Pella in Macedonia (as shown in Fig.1). The tomb is 10.30 m long, 6.70 m wide, 6.10 m high and it is dated to the end of 4th or the beginning of the 3rd century BCE (Chrysostomou, 2003, 145).

According to Haddad (1995,73-74; 1999,162-63), the tomb lateral walls thickness, which support the vault are about 1.50 m, whereas the thickness of the are just 0.65m. Several years later at the same place, Vergina, the lateral walls are constructed with thickness of approximately 1m at Philip' II tomb, then later on at the neighboring tomb of the "Prince" (last quarter of the fourth century BCE) are decreased to 0.55-0.60 m.

In the newly discovered unique four chambered Macedonian tomb at Amphipolis (span chamber approximately 4.5 m), the thickness of barrel vault and the lateral walls are approximately 0.62-0.65 cm. According to Peristeri, the head of Amphipolis tomb excavations, she dated the tomb to the late period of the 4th Century BCE, which is the period after the death of Alexander the Great (323 BCE). However, from the end of the fourth century BCE onwards, it is observed that in all of Macedonian tombs example, the thickness of all the walls, including the lateral ones and the vault are between 0.50-0.60 m thick. Thus, we can monitor several indications of the developmental phases in their construction.

In fact, the particular encased form and geometry of Eurydice barrel vault, imposed many question whatever it had a functional and decorative, structural or constructional role. What this indicate? Is the architect and the builders were probably not confident of the strength of the vault? Is the early Hellenistic architect who designed the tomb had no vision about strength?

Thus, the paper's scope should focus on the structural assessment of Eurydice tomb construction. Therefore, the aim of this study is not only to find answers to the debate, but mainly to shed light and understand if there was certain practical geometrical analysis technique applied to achieve the stability of this early Hellenistic barrel vault. This might rise the conflict of the Macedonian barrel vault debate to more sufficient debatable level.

In order to accomplish this task, a general historical and structural behavior study was carried out regarding the barrel vault and its geometry, by following the theories developed by recent researchers, in order to compare these results and to rationalize them in this unique example of Eurydice tomb. Mainly, to undertake a task, that till now, was never accomplished.

The paper will introduce the barrel vault engineering background by reviewing and analyzing methods developed to date, in addition to the geometrical and structural analysis that were conducted for Eurydice tomb. This methodological approach aims to investigate if the tomb geometrical and structural features exhibit actually structural integrity and strength that its behaviour remains always elastic. Thereby, demonstrating the ability of the Macedonian Greeks to construct the first extremely safe barrel vaulting structure.

The following sections attempt to investigate the architectural particularity, the exhibit geometrical and structural analyses of Eurydice tomb, which is considered as discussed above, as a critical case in the problematic chronology of the early Macedonian tombs.

In addition, an attempt is also done to evaluate and answer the mysterious existence of the encased vaulted tomb structure within the two rows of blocks above the vault crown area .

2.1 Particularity of Eurydice Tomb: Architecture and Structure

By contrast with most of the Macedonian tombs, with engaged order façade formation, the façade of Eurydice tomb is not architecturally developed, except the necessary functional marble doorway. It is only plastered with off white mortar, and covered by a second wall (Andronikos, 1987, 3–8).

The wealth of this tomb, which had been looted in antiquity, indicates a royal burial; the impressive and unique marble painted throne (2m high) is perfectly and richly carved and painted ornamentation. The back of the throne, depicts the "Rape of Persephone", where Pluto and Persephone riding on a Quadriga, meanwhile the throne feets are decorated with carved outstanding golden anthemia and helixes(as shown in Fig.2).

In this semi-cylindrical barrel vaulted double chambered structure of Eurydice tomb, the engaged order formation (shown in Figure 2) appeared on the tomb interior rear stone wall of the main chamber, which is prepared as if it was the exterior engaged façade of a Ionic building. In fact, the so-called "engaged order", in Greek architecture is evident that began as a decorative solution applied to the interior space, and consequently, transferred and developed radically at the exterior façades formation as can be seen at Philip II (336 BCE) façade in Vergina (Haddad, 2012) shown in Figure 3. This is in fact the case of the architectural formation conceptual approach of the earliest Macedonian tomb of Eurydice, whereas the façade has no formation.

The tomb main chamber back interior wall (shown in Figure 2) is decorated by a false façade of three bays divided by four engaged Ionic semi-columns attached to projecting pilasters; the central intercolumniation is decorated by a false Doric doorway, while the others are decorated by false Doric windows, which support an Ionic three-tiered entablature and a frieze decorated with white palmettes.

However, Eurydice tomb architectural formation with this unique interior engaged order, with the Ionic capitals, the false door and windows, suggest that the date of tomb is the same with the palace (350-340 BCE) (Koutaridi, 2011) and that is the existence of the barrel-vault itself.

As in all Macedonian tombs barrel vaults, Eurydice semi-cylindrical barrel vault is "Longitudinal vault", with the shape of laying half of cylinder cut along a horizontal plane. In longitudinal vaults the unit courses are parallel to the abutment development and joints are orthogonal to the correspondent element of the arch. A barrel vault covering a chamber can be considered as a longitudinally extended arch (Szolomicki and Berkowski,2013).

Masonry barrel vaults, generated by the translation of an arch (modulus) along an orthogonal directrix, as used in Eurydice tomb, are the simplest and oldest type of vaulted roof. The vault is built of porous stone, carried and supported on massive thick also porous stone walls (1.57m), where the doorways walls with the marble shutters is with a thickness of 0.65m.

As shown in the figure (5), the vault has a span of 4.5 m and a height is quite considerable 5,77m. It is of height that may causes structural difficulty at the level of the vault supports.

This barrel vault design set forth new challenges for this early Hellenistic structural engineering achievements; the more an arch is raised; the ticker the abutment has to be at the base.

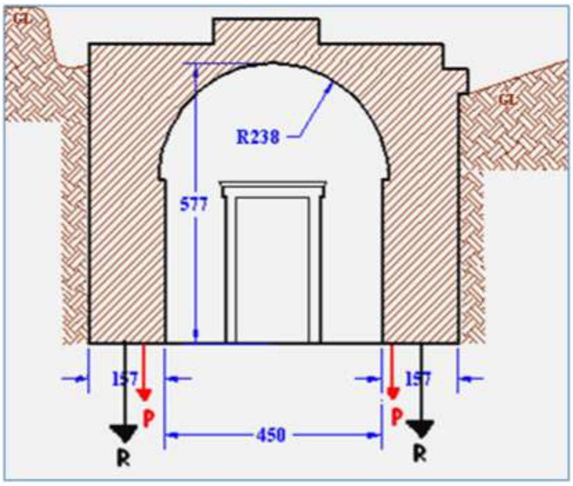

Figure 5. Cross section of Eurydice tomb geometry with the main dimensions.

3.MASONRY BARREL VAULTS: ASPECTS OF GEOMETRICAL CONSTRUCTION AND STRUCTURAL BEHAVIOUR

Technically, a barrel vault is formed when the arches of several conventional vaults are placed side by side, one after the other. The stresses within the vault are primarily compressive. A barrel vault, though, is a structural system that distributes loads by arch action, where the compressive stresses is transferred by means of a special arrangement of the building stones from the upper part of the vault to its sides, to the retaining wall and from there to the foundations and the ground. Therefore, the main problem with the construction is the generation of pressure on the lower parts of the vault and bearing walls. It dependents on four variables: the free span, thickness of the arch, the arc of embrasure, and the abutment thickness of the lateral walls.

A brief explanation of the different geometrical elements of the arch was set by Sondericker (1904, 118) in his book Static Graphics. As shown in (6):

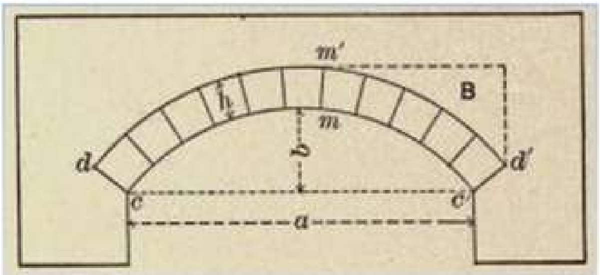

Figure 6. Definitions for the different parts of the arch (Sondericker 1904,Fig. 70, 118).

(*a*) is the span of the arch, (b) is the rise of the arch, the intrados (*cmc*') is the inner surface of the arch ring, the extrados (*dm*'*d*') is the outer surface, the crown is the highest part of the arch, the skewbacks are the surfaces (*cd*) and (*c*'*d*'), the haunches are the portions of the arch ring between the crown and skew-backs, the spandrel is the space(*B*)outside the extrados and within the dotted lines, and finally the backing is the masonry lying in space (*B*).

Due to the dissipation of the load forces to the sides of the vault, thrust is exerted on the building stones. The usual massive masonry barrel vaults load their substructure with considerable horizontal forces along the whole length of the walls, which is a very unfavorable influence on their masonry substructure (Bursié and Ferschin, 2003). Meanwhile masonry arch always push outwards, a barrel vault exerted a continuous thrust along its sides. Because some of the load creates lateral thrust, a balance of the static scheme is required along the sides of the vault. However, in order to build barrel vault, the walls needed to be extremely thick and devoid of any kind of apertures. Basically, thicker walls are made in order to absorb the load.

On the basis of recent results, it may be confirmed that, a good interaction among the arches composing the vault improves the load capacity(Huerta, 2001,p. 54). However, since the tensile strength of masonry is low, under certain loading conditions the vaults masonry is vulnerable to cracking and dangerous failure mechanisms (Taranu et al, 2010, 186-191). Nevertheless, cracks are not dangerous, but large unrestricted displacements of the abutments can lead to the catastrophic collapse of the structure (Huerta,2001,p.47, 54). Sliding is prevented by sufficiently high friction at the interface between the voussoirs, and by the resistance of the abutments to lateral force. If friction, though, was enough on its own, voussoirs would not have to be wedge-shaped.

According to (Szolomicki and Berkowski,3013), in the singly curved vaults principal stresses situated along the curve there are always compressive, and the pressure line system requires adequate weight of supporting system due to the relatively large forces of expansion. However, their behaviour is significantly affected by the low tensile strength of the component materials (Taranu et al, 2010, 190).

Heyman clarifies that a full semicylindrical arch must have a thickness of about 10% of its radius, if it is to contain such an inverted catenary. And, this required thickness reduces very rapidly if the arch embraces less than a semicircle (Heyman 2000). Usually the thrusts dissipated in the heavy mass of the haunching and the supporting walls. According to (Huerta,2001,p.57) the height/span ratio of a barrel vault determines the magnitude of the thrust (horizontal reaction force at the base).

Variations of rise-to-span-ratios have significant effect on the load-carrying capacity of vaults. The critical rise-to-span ratio is H/S = 0.2. In Eurydice tomb this ratio is 238/450= 0.52888. Though, the shallower the arch, the greater is the horizontal thrust. Of significance that, the presence of horizontal thrusts, however, was already perceived at least in the 1st century BCE, as reported by Vitruvius within *De Architectura,* (Benvenuto, 1981,1991).

3.1 Conditions of Structural Stability of Stone Barrel Vaults

As mentioned, due the heavy semi circular stone arch, an outward force is created in the lower portions which lead to the collapse of the vault. Lateral stability is developed within the plane of the vault, due to its continuous form (Unay, 2000). However, there are three aspects to be considered, which are deeply interconnected: geometry, construction and stability. The geometry plays a key role in the structural behaviour of shell structures (Arnout, et al., 2010,1). Finding the optimal shell geometry is therefore of crucial importance.

Though, to assist the structural safety of a barrel vaulted structure that function well, there are two design principles/conditions that ensure the equilibrium are guaranteed; to design arches that will stand, and buttresses which resist their thrust. These are crucial for the stability that guide the design. However, the most critical problem is the buttresses because it involves the collapse of the whole structure (Huerta, 2001,50).

Generall, (Heyman, 1995) conclude that, any vault will stand as long as a thrust network can be found that fits within its voussoir's thickness section. The collapse of a masonry arch occurs when the load path can no longer be contained within the masonry. The vault, though, are shaped so that its cross sections is only under compressive stresses, which correspond to its geometry, and the conduct of the pressure/ thrust line dependent on the way of loading and boundary conditions (Szolomicki and Berkowski,2013).

Though, the main requirement is that the material must act in compression, i.e., in every section, the thrust (stress resultant) must be contained within the masonry. Figure 7 shows a schematic illustrations of the minimum and maximum possible thrust lines in a semicircular arch under its own weight. In fact, the line of thrust (line of pressure), in a masonry arch, produced an enormous advance in understanding of arch design and analysis[3], while it represents a possible static state of equilibrium (in compression) (Huerta, 2008, 325); if the line is contained within the masonry section (voussoir's thickness), the yield condition of the material is satisfied, and the arch is safe. The form of the line of thrust depends on the geometry of the arch, its loads and, also, on the family of plane joints considered (Huerta, 2001, 57; 2008).

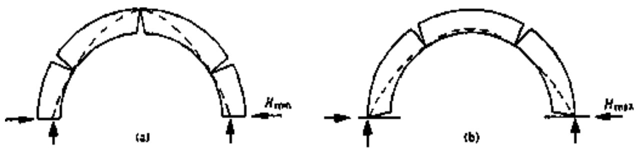

Figure7. Thrust lines in the semicircular arch: a) Minimum thrust; b) maximum thrust (After Heyman, 1995).

However, any little movement of the abutments will produce a certain cracking and a change in the position of the line of thrust, but due to the Safe Theorem it will never go out of the masonry (Huerta, 2009). In the semicircular masonry arch the line of pressure does not conform to the shape of the arch and therefore the crown of the vault tends to fall while the sides buckle out. And the determination of the collapse state can be defined based on thrust line analysis using graphic statics (Huerta, 2008, 305)[4].

On the other hand, when the line of thrust touches the limit of the masonry a "hinge" forms, which allows the rotation. Hinges form where the thrust line encroaches the structure boundaries and masonry cracks appear(as shown in Fig.7).Three hinges make the arch statically determined and, an arch with three hinges is a stable structure.

However, an increase of the load will lead to the formation of four hinges and will lead to collapse without crushing of the material. This can occur in a stable arch with addition of load, which deforms sufficiently the line of thrust (Huerta,2001,56). Alternatively, a collapse mechanism can be imposed on the arch, and the load associated with this collapse mechanism can be calculated, recognizing that this load is an upper bound on the collapse load (Boothby et al., 2006).

3.2 *General Chronological Critical Review of the Main Theories of Vaulted Masonry Structures Stability*

Various analytical and empirical methods for the analysis of masonry vaults are available in literature (Romano and Grande, 2008; Huerta,2001, 2005, 2008). However, although the history of the theory of the masonry arch is quite well known, the development of the theory of spatial masonry vaults is only roughly known (Huerta, 2008, 307).

For example, Leonardo da Vinci equilibrium condition of an arch comes from a sketch in his *Codici di Madrid*, clearly illustrate his idea for the structural model of each elemental arch, derived from the geometrical decomposition of the vault; "the arch will non crack if the chord of the outer arch will not touch the inner arch" (Galassi et al., 2012,134).

The plasticity methods, first applied to medieval structures by Heyman aimed to understanding of the behaviour of masonry arches and vaults. For example, Gothic architects used their medieval rules of construction -and Vitruvian rules- which were concerned with geometry and with correct proportion (Fangary, 2010). They also understood the necessity of buttresses or abutments, so they designed it by numerical rules of proportion (Heyman, 2000).

From the end of the seventeenth century a "scientific" theory of vaulted masonry structures has been developed (Benvenuto, 1981; Heyman, 1995; 2000). Of significance is Heyman (2000) about the seventeenth-century statements of Hooke/Gregory to understand the phenomena; "if a position of the catenary can be found which lies within the boundaries of the masonry, then the structure is satisfactory" (Heyman 2000). Also Rankine (1858) proposition was enounce by a significant theorem; the stability of an arch is secure, if a line of thrust, balanced under the forces which act on the real arch, can be drawn within the middle third of the depth of the arch ring. Indeed, if the thrust were contained within the middle-third of the section, there was no possibility of forming any cracks[5](Huerta,2008, 305).

However, it is impossible to know the actual line of thrust, but this is not essential, as we can calculate the safety of the structure without making assumptions about its actual state (Huerta,2001,56). Nevertheless, the Safe Theorem of Limit Analysis solves the problem of finding the actual line of thrust.

By 1900, the graphical analysis as practical approach was the standard tool to check the stability of any arch, meanwhile elastic analysis was employed as a final check in the most important bridges (Huerta,2008, 305).

The best approach as it was supposed to permit to know the position of the true line of thrust, and therefore to know the actual internal forces and stresses in the arch was considered. However, in practice the usual procedure was to check the stability by graphical statics. Though, in past behavior of masonry structure is modeled by classical methods like graphic analysis (Boothby et al, 2006).

Heyman (1995) has incorporated the old theory of masonry structures within the broader frame of modern Limit Analysis. Huerta (2005) proposed the use of Tilt Analysis for the study of arches with simple plane block voussuoir models. The application of the limit theorems of plasticity to historical masonry vault constructions, formalized by Heyman (1995), are particularly suited to fulfill the method's three base assumptions: material has no tensile strength, but the compression strength of the masonry is unlimited, meanwhile sliding between the different components is impossible.

This rational theory was preceded by the traditional "geometrical" theory of the old master builders. Both theories which tried to solve the fundamental problem of how to design safe vault structures, arrived to same conclusion; the safety of a masonry structure is a matter of geometry, while a safe state of equilibrium is achieved through a correct geometry.

Though, both historically and theoretically the "equilibrium approach" is the best approach to the analysis and design of masonry vaulted structures (Huerta, 2001, 47).

Actually the structural analysis focuses on the geometry of the structures rather than strength, while the Limit Analysis, developed mainly by Heyman 1995 enabled to shift the problem from calculation of the structure's stress state to the definition of possible collapse mechanisms. However, by the growing interest in the preservation of historic structures, studies for method analysis of load bearing unreinforced masonry structures, such as arches, vaults, and buttresses were developed for appropriate solutions in the assessment of structural performance (Unay, 2000).

Recently, progress has been made in the development of constitutive laws for ancient masonry structures and in the application of these to the analysis of unreinforced masonry structural systems (Boothby, 2001, 246).

Recent results of analysis indicate that the geometry, initial geometric imperfections, type of loading, unsymmetrical load distribution, boundary conditions and material yield stress have considerable effect on the type of collapse mechanism, load carrying capacity and structural stiffness of studied systems (Mohammadi et al., 2012, 217-218).

Finally, computer programs are of a great help. The development of a computer method is of vast importance, with regard to strengthening interventions; it allows understanding the cause of the cracks initiation and its propagation (Szolomicki and Berkowski, 2013; Huerta, 2008,325). However, only few computer packages are available to help the analyst in the study of the equilibrium of arches, vaults and buttresses.

4. DISCUSSION OF THE GEOMETRICAL, STRUCTURAL AND COMPUTING RESULTS

Development of structural concepts, and not the changing of fashion in decorative forms, is the driving force of the history of architecture (Bursié and Ferschin, 2003, 1855-65). In fact, history of architecture is inextricably linked with the history of construction techniques.

According to the classical canon, the geometry and the functionality of a building's load-bearing elements and structures effectively specify its form (Kottaridi, 2011). Though, the architects ideas of space and form cannot be materialized without understanding and controlling the construction behavior of a structure.

In the case of masonry buildings, the geometry is much more complex; it is more difficult to produce programs easy to use and which may be adapted to the building in question. However, as shown, the strength of a masonry arch depends primarily on its geometry, while in the study of the collapse of masonry arches, the fracture of the abutments should be considered. This can result in lower collapse loads for buttressed arches, meanwhile the fracture of the abutments reduces the collapse load by approximately 14% (Ochsendorf et al., 2004, 95-96).

On the other hand, the processes of estimation which made it possible for the early Hellenistic architects to work out the size of arch/ barrel vault are unknown. However, the structural form and the intrinsic weakness of porous stone to tension can produce fracturing at the intrados of the crown accompanied by large thrusts at the springing of the vault with the potential for catastrophic collapse.

The method of erecting Eurydice tomb vault had been a mystery, while the way in which the architect of the tomb used the stone for building the vault is unique. Furthermore, it is unfeasible to be sure how the details of the vault were actually made; the vault surface is only visible from the interior and not from the exterior.

There is no information even for using uniform metal joints, that subsequently contribute to link the stones flattening the contact surfaces and distributing stresses uniformly as it is evidence in other early Macedonian tombs.

The 'Π' shaped iron nails in intervals over the entire surface of the vault were probably not used for structural and stability aims, but for decoration and aesthetic purposes, in order to fix and carry a light wooden beams structure, which had been covered with fabric, as indicated from the plethora organic materials found in the tomb floor, and even from the examples of the later Alexandrian Ptolemaic barrel vault tomb roofing decoration.

Actually, the structural form of Eurydice tomb can simply be defined as the geometrical configuration of the space involved by the structure. There are many factors that determine the bearing capacity of Eurydice tomb vault such as its shape, its span and its thickness, but also the mastery of the placing. At the level of its supports arch brings about an oblique thrust, which is the result "R", of each stone voussoirs, in order to reabsorb this thrust, the builder thus has to create a mass, the vertical weight of which "P" is the greater than the sum of the thrust.

Certainty a simple balance of forces is not enough, for its necessary to take in to account external constraints such as settling of the foundation, soil pressure and various forces acting upon the building.

4.1 Geometrical design rule analysis of the barrel vault and buttressed bearing walls of Eurydice tomb

In opposite of many Macedonian tombs under Tumulus, the stability of Eurydice tomb structure depends mainly on the self weight. This while the original ground level (GL) is approximately the same with the upper stone level of the tomb barrel vault (shown in Figure 9).

The architect had in his mind to make sure that the weight of the wall is larger than for the arch and decided to increase the thickness of the wall, so that the reaction due to the wall will not affected by the eccentric (lateral) load from the barrel vault. Thus, by increasing the thickness of the bearing walls (1.57 m), he decrease the effect of shear force (distribution will be on larger area) on the supports, while the internal shear stresses will be larger.

Geometrically, the unordinary thickness of the wall buttress of Eurydice tomb is in fact of a certain fraction of the vault's span. Interesting enough for our study of the geometry of the barrel vault of Eurydice tomb, is Viollet-le-Duc (1858) formula for the determination of the strength needed for the masonry shoulder to withstand the thrust of an arch of vault (as shown in Fig.8).

He related an extremely simple geometric formula to determine the thickness of the buttress bearing wall/ columns in relation to the thrust to be carried (Antonioni et. al., 2007; Fangary, 2010, 12). Actually, From the Roman period to the Renaissance, the problem of stability and correct design of arches was mainly faced under this geometrical point of view.

The formula basically consists of dividing a half-circle of the intrados into three equal parts (cords); the direction of the thrust will be given with sufficient accuracy by the direction of two lateral segments beyond the springing line (As shown in the Fig. 8).

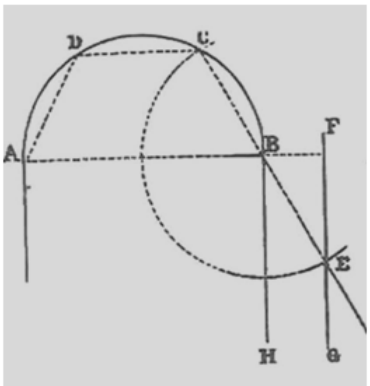

Figure 8. Geometrical design rule still used during the 18th century for the determination of the strength needed for the masonry shoulder to withstand the thrust of an arch of vault (Viollet-le-Duc after Fangary, 2010, Figure 12 and Antonioni et al, 2007).

This geometrical design rule/ formula was still in used during the 18th century (Benvenuto, 1981). Huerta explained this old traditional rule for the design of masonry vaults and buttresses confirming that, they define certain proportions between the structural elements (Huerta 2001). However, experimental analysis of this method proofed that, it provide the structure with the required stability to keep it standing over thousands of years(Fangary, 2010,12).

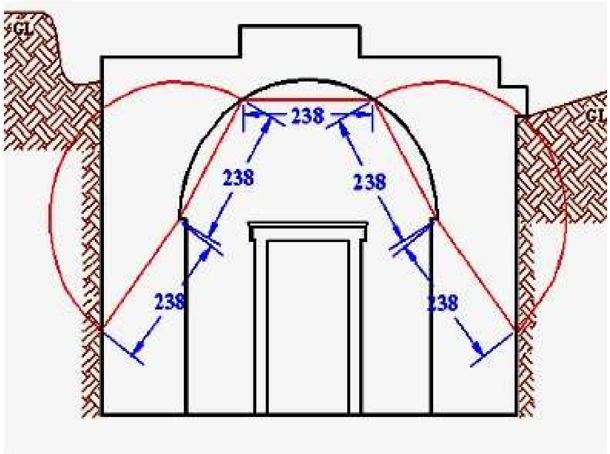

Figure 9. Graphical analysis of Eurydice tomb vault geometrical design rule for determination of the strength needed for the masonry shoulder to withstand the thrust of an arch of vault.

As shown in Figure (9) this method/ formula of estimating abutment masses, was already known and was applied in Eurydice tomb modeling in Macedonia since the early Hellenistic period, around 340 BCE, before even the Romans to guarantee the stability of their arches/ vaults. Though, the thrusts are be kept within the wall section, and the line of thrust is within the encased solid external wall. As the main force is due to self weight, the forces along the vault balance one another that is until the end. At the ends are more solid constructions, the main two external wall and the weight of these resist the end thrusts. Tough, the significant of our attempt to decode the relation of the parts of the building point to the intelligence of its design and the accuracy of its execution.

Indeed, since the time of Masters, empirical approaches and structural intuition constitute a fundamental basis for the interpretation of load transmission mechanism of historical structures (Unay,2000).

This graphic procedure, documented since the renaissance, permits a very satisfactory estimation of the size of the abutments of the voussoirs arches. Though, we can argue that, this is the case of a practice that has been handed down in the most commonplace way by traditional and professional teaching since the early Hellenistic period.

In conclusion, we can assume the rule for the early Hellenistic design of masonry barrel vault and bearing wall/ buttresses is geometrical, in the sense that it define certain proportions between the structural elements (the thickness of the buttress is a certain fraction of the vault's span).Therefore, we can argue that, Eurydice tomb is an affirmation that we can find the first Greek essence of the structural barrel vault design of masonry. From the early Hellenistic, though, to the Renaissance time of the barrel vault construction, the masons' knowledge was only related to the geometry and proportions.

4.2 Result and discussion methods of computing geometry

As mentioned, stone is extremely resistant to compressive stresses, while its tensile strength is much 1ower. Therefore, when used for covering spaces, it is applied as massive masonry vaults, which had a shape adjusted to the pressure/ thrust line, in order to avoid tensile stress. In fact, masonry structures have even lower tensile strength than stone as a material, because the joints between stone elements have a negligible tensile strength (Bursié and Ferschin, 2003,1859; Di Pasquale, 1984). The structural resistance depends primarily on two factors: the geometry of the structure and the characteristic strength and stiffness of the material used (Ochsendorf et al., 2004,88). A structure, however, will never, in practice, fit perfectly to its foundations. In general the foundations are not perfectly flexible and are embedded at a certain depth below the ground surface if the foundation is subjected to a uniformly distributed load, the contact pressure will be uniform and the foundation will experience a sagging profile. So, in the analysis we will consider the foundation rigid.

4.2.1 Structural analysis and material

A conservative estimate was made for the material properties using values derived for the porous limestone. "γ" is the unit weight of material for specific volume and it differs from one material to another, and the unit of it is (kN/m^3), so we will use it to change the unit weight of soil and stone to load acted on structure.

Eurydice tomb stone blocks has a different dimension. They range from 46 cm to 53 cm, so we will take the average of all blocks approximately equal to 50 cm. The lime stone block will be considered as (50x50x50 cm^3 block dimension),and γ_{stone} =14 kN/m^3. Eurydice tomb supported on stone below each abutments for many structural reasons, the most important is to prevent differential settlement, so cracks will not happened. Structure supports on a clayey soil under stone layer and it fills from two sides, so the walls act as a retaining wall, and γ_{clay} = 18kN/m^3.

As shown in Figure**(**10)**,** the supports are assumed to be fixed between soil and bottom tomb. In the left side the soil is filled up to 554 cm. Pressure will be varied from maximum (γH = 5.54×18 =99.72 kN/m.) at the base, to zero at the top. In the right side the soil is filled up to 493cm. Pressure will be varied from maximum (γH = 4.93×18 =88.74 kN/m) at the base, to zero at the top. Where H is the height of the fill behind the wall (m). Because of the difference in fill between left and right side, the moment, shear, and deflection diagram is different in the two sides. In the upper flat surface of the tomb an additional two stones block are placed above the vault crown area in the keystones area, so it's an additional load on the vault (load =14×1.0×0.5 = 7.0 kN/m).

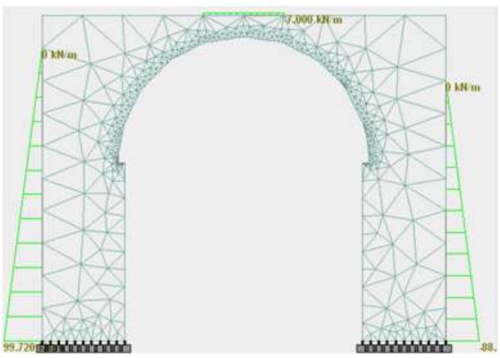

Figure 10. Eurydice tomb structural analysis of Load values.

4.2.2 Strength and behavior of prototype of Eurydice barrel vault in the original conditions

To assess the behavior of Eurydice vault and loading conditions. These conditions have been suitably simulated and applied with STAAD Pro, which is a structural analysis and design computer program and considered as good software for analyzing arches-curved beams whi8ch frame the elements used in Eurydice tomb, as also it is easy to deal with all types of soil for selecting foundations. An important issue also is that, it provides a deflection diagram for the structure and weakness point. An analysis has been done to find the shear and moment diagram and the deflection diagram (as shown in Figs, 11,12,13). The tests on the model barrel vault and the analysis showed that, the vault has a significant degree of strength. The results are shown below.

In Fig. 10**,** the load diagram is shown, considering the self weight of the structure and the soil pressure which distributed as triangular shape. Fig. 11 shows the shear force diagram; it is clearly that the lateral force from the soil prussre act at an opposet direction of the load from the arch, which reduce the horizantal internal forses so that *fx* will be less. Fig. 12 shows the bending moment diagram, the maximum moment on the supports drown on the tension side on the wall. The maximum at 0.6 h moment change it sign up to h. The moment on the arch is small and the magnitude increase with the thickness.

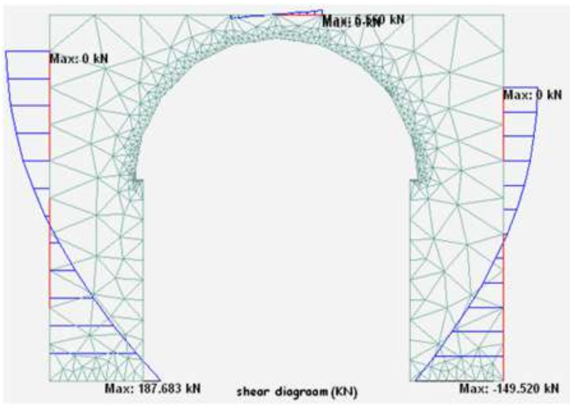

Figure 11. Eurydice tomb structural analysis of shear diagram.

As known, to every possible movement corresponds a certain cracking, and cracks open and close to permit the arch to respond to this aggression of the environment.

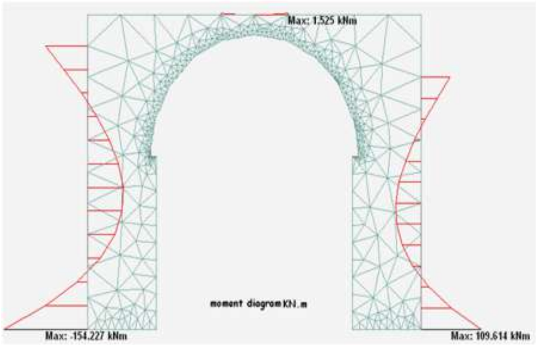

Figure12. Eurydice tomb structural analysis of moment diagram.

However if the movement is small (say 1/100 of the span, or 100 mm for a 10 m span), the same state of equilibrium will be contained within the distorted geometry; in fact, 1/100th of the span is more or less the thickness of the lines of the drawing (Huerta, 2008, 325).

The deflection diagram in Fig. 13 shows that the frame will sway to the right because of the soil pressure (active soil pressure), also it gives indication about the mode of failure. Therefore , it is recommended to fill again this part of the structure, as it was in the original conditions before the excavation.

It seems that, in Eurydice tomb structure nothing is left to chance. The results of the analysis shows that the structural skeleton of the tomb, characterized by the innovative use of shear walls, and supporting porous limestone blocks, can be regarded as the earliest prototype of the structural solution adopted for the design of this early Hellenistic of massive stone vaults.

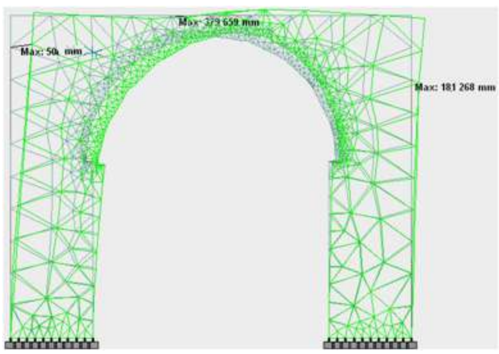

Figure 13. Eurydice tomb structural analysis of Deflection diagram

The marvel of this tomb structure is how the architect, without concepts of stress and moments were able to balance the various thrusts and keep them within the funicular lines.

4.3 Investigation of Longitudinal Vault of Eurydice Tomb

The pushing that characterizes vault structures, makes them basically prone to instability/damage, propensity that can be enhanced by any seismic phenomena. Recent result underlines the good response of the longitudinal vault, which is the case of Eurydice tomb, under seismic loads compared to the transversal one[6] (Romano and Grande, 2008).In addition, the longitudinal arrangement, although less stiff of the transversal one, is able to carry a bigger peak load due to the skew of each course with the adjacent ones.

For vertical concentrated loads, the behaviour in terms of strength of the longitudinal vault is also better than the Transversal one. The latter, although stiffer, is influenced by the behaviour of the single arches which act almost independently. The longitudinal pattern, conversely, involves the collaboration of the courses in the nearby of the loaded part.

The Longitudinal vault, in the case of increasing horizontal loads, also behaved better than the Transversal one, being able to bear a bigger peak load. Even in case of concentrated or horizontal loads, the skew of unit courses like the Longitudinal vault could make the difference, increasing consistently the load capacity of these structures. This while the skew of the blocks furnishes a further contribution to the global capacity which also depends on the type of the load condition.

4.4 Investigation of Bearing Wall /Buttresses of the Barrel Vault of Eurydice Tomb

The masonry buttress in Eurydice tomb is a series of individual stones placed roughly in horizontal courses. In fact, the safety of masonry vaulted structures depends on the stability of the "buttresses", which is perfectly designed as shown in Figs (5, 8,9,11,12). To support the thrust of tomb vault including the fill over the extrados, the overturning moment produced by the thrust of the vault equilibrated by the mass of the buttress.

Figure 14a shows the collapse of a vault supported on masonry buttresses. Collapse of a vault supported on masonry buttresses, however, postulates a fracture in the buttress along line *XK*. A line of thrust is approximately shown, presenting its relationship with the assumed fracture (Cain 1879, Ochsendorf, et al ,2004, 90). Meanwhile, figure 14b is showing the line of thrust in buttress formed by horizontal courses of masonry where the collapse load *F* will be less than the overturning load for the solid buttress *Fs* (Ochsendorf, et al ,2004, 89).

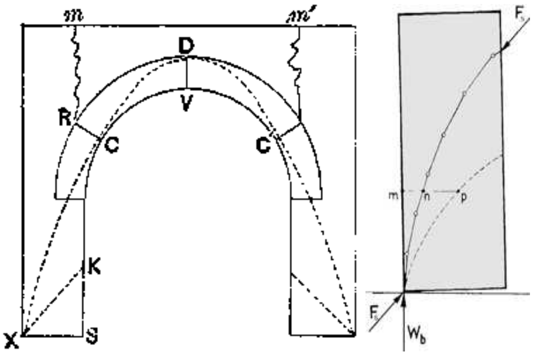

Figure 14a(left). Collapse of a vault supported on masonry buttresses (after Cain 1879). FIG.14b (right). The position of the line of thrust in buttress formed by horizontal courses of masonry (After Ochsendorf, et al ,2004, Fig. 1.b, p. 89).

4.5 Investigation of the Massive Stone Spandrel Filling at the Barrel Vault Extrados of Eurydice Tomb

Many researchers accredited the use of massive filling introduced at the haunching of the vault with the heavy walls to be sufficient to restrain the extensive thrust generated by a relatively thick structure (Acland, 1972; Fangary, 2010, 15),which is the case of Eurydice tomb. Moreover, on the basis of recent results of studies and analysis, it was found that the spandrel filling material at the vault extrados, influenced the mechanisms layout occurred.

In fact, the fill of the haunches add further to the stability of the construction (Heyman, 1995, 51). This while the horizontal to vertical thrust ratio at collapse is lower in the arch with fill, hence its cracking acceleration is higher. The spandrel walls, though, over the chamber vault can improve the balance between the loads and reduce the deformations.

However, earth pressures from the spandrel fill can be significant only when the fill reaches an excessive height (i.e., the apex of the vault). Recent investigations also of the vault behaviour without fill suggested that the system would have failed with an alternative mechanism and under reduced lateral acceleration (Perugini ,2013,14).

4.6 Investigation of the Function of the Two Rows of Blocks Over the Crown Vault Area

A higher vault thickness as a significant part of the weight of the upper vault structure is carried directly over the crown area of the key stones, thus increasing the deflections in that area of the vault. Logically, combined with movement of the abutments, this loading should caused failure. For that, in the next phase of the test structural analysis, the assessment of the response of the vault to service loads was concluded with increasing this fill. No significant change was recorded in the deformation of the model vault in these conditions. If we consider the upper two blocks acts similarly as in the case of the Renaissance dome lantern at the crown area, running the analysis again, there is no series change occur (just for the self weight). As mentioned, an arch is considered safe if it is possible to draw a thrust line within its middle third, as this is the geometrical condition that leads to a distribution of compressive stresses in all the section. The line (surface of thrust) moves upwards looking for the minimum thrust position. For example, if the dome has no lantern, as a small dome-cap supported by a series of radial arches, the line does not touch the extrados at the crown, but at some distance; the upper part, then, remain un cracked, (Huerta, 2008.323).

Interesting enough are also the results of the recent study with Graphic statics analysis of Gothic vaults (Fangary, 2010,vii), about the significant role of the infill layers in the equilibrium of the vault and the transmission of the lateral thrusts; for most of the analyzed arches, it was not possible to find a line of thrust within the stone voussoirs region and the line was passing either to the sound or light infill layers. However, it was not possible to find a realistic line of thrust for the cross ribs arches when the loads of the webs and the upper infill were applied to them. Actually the minimum and maximum values of the lateral thrusts for the arches were almost the same, and the cross ribs mostly don't have a structural role (Fangary, 2010). This might explain why the architect of Eurydice tomb decided to design the tomb extrados with spandrel filling including the filling the above crown area of the vault with the two rows of blocks.

In conclusion, till now, it was not possible to define structural theories applied by the early Hellenistic masons, and how the stability of the structure was achieved. This unique barrel vaulting system confirm high skills of the Macedonian masons in the time around 340 BCE. Everything is subjected to the "equilibrium of measure". This archetypal building, designed might be the logical proposal of the "ideal equilibrium structure", at the culminating point of ancient Aegae. The example of Eurydice tomb was radically new, original constructive and its structural concept, broke with the tradition used in the east.

We can assume that, Eurydice tomb design, as the case of Vergina/ Aegae palace, are of the most structurally astonishing buildings in Macedonian history. It revered for the buildings use of the first large barrel vaults and referred to as one of the greatest engineering achievements of Macedonian Aegae. Though, we can suppose that, the geometrical and mathematical skills of the architects by the beginning of the second half of the fourth century BCE, allowed them to layout the barrel vault over oblong compartments with rational accuracy. The design of this early Hellenistic barrel vault structure of Eurydice tomb, in fact, followed grand rules of geometry which were transmitted through the practicing masons. It can be assumed that the Macedonians mostly learned from their success more from their failure. However, one should examine if these rules were kept as kind of secrets and were forbidden to the public, as in the Gothic and Renaissance period, where scraps of this knowledge were deduced from some sketchbooks and texts originating from late Gothic when some relaxation of the regulations took place (Fangary, 2010,12).

Finally, according to Kottaridi (2011), the architect of Aegae palace(ca. 12,500 m^2) who designed and created this edifice, the greatest building in classical Greece "was an ingenious mathematician but at the same time a daring artist and innovative theorist... Using the means which the king set at his disposal, he opened new paths for future architecture"(Kottaridi, 2011,331).

The recognized architect *Leochares* might put his chisel at the service of Phillip II. It is probable then, that *Leochares* was collaborated with *Pytheos* and who had already been proved in his great innovative work for Mausolus. This suggestion is worth further study.

5. SUMMARY AND CONCLUSION REMARKS

The Macedonian tombs barrel vault structure is a fundamental feature of Hellenistic architecture and its development. Roofs based on the compression principle were used in Greek architecture at least mostly around 340 BCE, and the earliest examples were started by Macedonian tombs. The barrel vault masonry of the earliest Macedonian tomb of Eurydice (around 340 BC) open new horizons to the later examples concerning the structural behaviour of the whole construction both toward vertical and horizontal loads.

From the early Hellenistic period, the architect of Eurydice tomb applied in his structure barrel vault, because he had the knowledge and could control and take vertical load and distributed it in a better way; in his structure he depend on calculating the thickness of the lateral walls to make it more rigid.

The architect developed certain geometrical method of structural analysis, and the behavior of the vault represented in detail by the design geometry of the vault. Many conclusions can be derived from this study, regarding the form and geometry, the graphical analysis method of the of the earliest Hellenistic barrel vault.

Consequently, in Hellenistic and Roman architecture, the problem of stability and correct design of barrel vaults were mainly faced and solved under a geometrical point of view, as shown in Fig. 5. This approach was already known and applied from the early Hellenistic period, and not form the Roman period, and which continued in the Renaissance until the 18th cent. This geometric construction formula was the key secret to understand as better as possible the structural behaviour of a voussuoir barrel vault. Eurydice tomb actually is one of these exceptional buildings with its unique barrel vault, constructed in an original way and different from the eastern parallel examples, which consequently results in specific structural behaviour.

The example of Eurydice barrel vault also suggests that, ancient Macedonian architects and masters had the knowledge of the basic theoretical laws of barrel vault structural mechanics. In fact we have to accept that, in Macedonia, the theoretical concept of structural force were developed at least by the beginning of second half of the fourth century BCE. For that reason, Eurydice Tomb is outstanding building, which made a giant leap in the history of Greek barrel vaulting construction with its original architectural, constructional and structural concept. This building, with its innovative concept proved to be structurally sound, became the origin of a new tradition of the development of stone barrel vaulting in Macedonian Hellenistic architecture.

The intention is to make clear that there was an old tradition of scientific estimation of barrel vault masonry structures, using similar approach of equilibrium. There is much to learn from the architects/engineers of the early Hellenistic period. They may not have had a perfect grasp of the "stone barrel vault equilibrium theory", but they do have the essential knowledge, which supply the practice. The first results from the geometrical and structural analysis of the barrel vault in early Hellenistic architecture, based on Eurydice tomb, has highlighted some of the structural aspects of the innovations in Macedonian construction technology. This stunning successes of the early Hellenistic architects of the vaults' safety factors could not have been a matter of chance.

The development of the Macedonian tomb semi-cylindrical barrel vault structures suggest that, a great architect dealt perfectly with the mechanical behaviour of barrel vaulting structures. In fact, his knowledge was not only truly experimental, he as an ingenious mathematician understood the mechanical behaviour, possible weaknesses or occurrence of cracks, and applied this experience to the structure that he was actually building. This while, as the construction process was relatively very fast, he was able to observe his own barrel vault building, its behaviour and weak points, and to intervene if necessary.

In conclusion, the geometrical and structural analysis of Eurydice tomb showed that, the designer engineer/architect was moving in the correct direction, formulating a new arc structural scheme that, once perfected, would have made possible the construction of large vaults such as those at Phillip II tomb, which can be considered the real existing building with its full scale model after Eurydice tomb.

The structure of Eurydice tomb with its encased vaulting and buttressing system, shows how early Hellenistic architects utilized certain techniques, that had been formed and refined over time and implemented them successfully on a scale that went unmatched. In the interest of modern research, it is the size in relation to the height and the width of the building, that makes it a target for answering questions about early Hellenistic barrel vault construction.

While there is still more research to be done, what has been found from studying the geometry and structure of Eurydice tomb has served as a first step in understanding of the early Hellenistic barrel vaulting buildings. More research for structural and geometrical analysis are needed for the other different examples and typologies of the Macedonian barrel vaults, in order to reach a final conclusion about their structural stability development. However, the results of this research can provide a good perceptive about the problem and can contribute to any efforts that will be undertaken for the conservation of such notable structural and architectural achievement.

Issues of durability of the masonry should also be examined, and once the effect of the major actions is better understood through analysis, aspects of design can be discussed by studying the incidence of variations in the layout plans, sections, and proportions of the Macedonian tombs barrel vaults.

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1 Based on chronological data, Eurydice Tomb is attributed to Philip's mother "Eurydice", inscribed dedications of whom have been found in the temple of Eucleia at *Aigai*, approximately 200 m north from the tomb.

2 After her arguments concerning the barrel-vault and the diadem had been answered by W. M. Calder III and E. A. Fredricksmeyer, Phyllis Lehmann attempted a rebuttal, which convinced neither of these two scholars, who maintained that there was no valid argument excluding the possibility of the barrel-vaulted tomb and the «diadem» at the ancient Macedonian capital in 336 BCE. See Fredricksmeyer, Once more the Diadem and Barrel-Vault at Vergina, *American Journal of Archaeology* Vol.87 (1983) 99-102; W. M. Calder III, Golden Diadems again , *American Journal of Archaeology,*Vol. 87 (1983), 102-103.

3 Therefore, if it is possible to draw a line of thrust (equilibrium) within the tomb arch (no-tension material), this is an absolute proof that the arch is stable and that collapse will not occur for the given loading (Huerta,2008,320-21). (Huerta,2001,p.56, Figure 11).

4 The aim of the use of the line of pressure (line of thrust) is to define whatever the arch is stable or not, meaning that the arch can support the subjected loads and its own weight. The location of the thrust line can be estimated using simple calculations of plane forces under arch loading due to self weight, superimposed dead load and live load. Therefore, equilibrium in a masonry arch can be visualized with a line of thrust. See Huerta,2001,56).

5 However, the impossibility of the existence of another line of thrust leading to a collapse mechanism was not demonstrated (Huerta,2008 , 305) .

6 In the Transversal Vault the stones have the major side orthogonal to the abutments joint are in parallel planes. The vault consists of many independent arches, leaning one upon another. Each arch is linked to the adjacent ones only by mortar joints. To avoid lateral displacements and saving centrings, it may be built also along inclined planes. The construction starts on the fronts and go on along the axis with a sliding centring (Romano and Grande, 2008).