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ON THE VALUE AND MEANING OF PROCLUS' PERFECT YEAR

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ABSTRACT

An 18-digit number for the length of the Perfect Year was given by Proclus in his commentary on Plato's "Republic". The number was corrupted in the manuscript tradition and now it is known up to some missing and uncertain digits. Previous attempts to reconstruct Proclus' value as a multiple of planetary periods known in ancient Babylon and Egypt led to number which has nothing in common in its writing with the one of Proclus, except the scale.

Our approach to finding the original Proclus' value is based on the assumptions that it should include as multiples some of Babylonian and Egyptian planetary periods and that the other prime factors should be of same (or at least comparable) scale as the planetary periods, known to the ancients. The value could contain also other periods, in particular, of Greek origin.

After careful examination of all possible candidate numbers for the original Proclus' value of the Perfect Year only one among them is shown to satisfy the assumptions.

At least one of them can be related also to metaphysical concept (the significance assigned to amicable numbers 220 and 284 by the Pythagorean tradition), which is in agreement with Proclus' method to calculate the Perfect Number. Another multiple, with difference of one year, appears as time interval, derived from the Turin Royal Canon, including its mythological columns, which may be also considered as confirmation of its significance for the ancient astronomers.

KEYWORDS: Great Year, prime factor decomposition, Sothic period, Goal-Year periods, amicable numbers.

1. INTRODUCTION

The concept of Great Year or Perfect Year can be traced back in time at least to Plato. It was generally understood as the period of return of the Sun, the Moon and the planets to their initial positions related to the fixed stars on the celestial sphere (de Callatay, 1996). Wide range of values appears in the ancient literature for the length of the Great Year. Oenopides of Chios considered Great Year of 59 years only (Evans, 1998), while other authors mention values of more than 10 000 years (de Callatay, 1996).. The usual algorithm for obtaining the value of the Great Year is to compute the least common multiple of the periods, corresponding to individual celestial bodies. Hence, the value of the Great Year will depend on the accuracy of the periods chosen.

There is, however, one value, which is extremely large, compared to others, and it is still unclear how exactly it was computed. This is the length of the Perfect Year which is given by Proclus Diadochus (c.411-485 AD) in his commentary on Plato's "Republic". The number was corrupted in the manuscript tradition and is now known up to some missing and uncertain digits, as follows:

$$N(x,y) = 344\ 300\ 787\ 638\ 360\ 600\ + 10^{12}x + y,$$

where x is two-digit integer not containing any zeros, and y equals 9 or 70. According to Proclus this number was computed by Sosigenes of Alexandria "from the returns to the same point of the seven spheres" (Kroll, 1901).

Historically, the Perfect Year of Proclus has been an object of many speculations and attracted the attention of eminent figures like the great Irish poet W.B. Yates (Jeffares, 1989). Despite this interest, not much progress has been achieved in the deciphering of this huge number.

The only existing so far complete reconstruction of Proclus' value (Neugebauer, 1975) is a multiplication of the "greatest returns" of the planets (265 years for Saturn, 427 years for Jupiter, 284 years for Mars, 1151 years for Venus, and 480 years for Mercury), together with Egyptian luni-solar cycle of 25 years and the Sothic period of 1461 years. The result, however, has nothing in common with the number given by Proclus except the scale – Neugebauer's reconstructed value has the same number of digits as the number provided by Proclus (de Callatay, 1996). The suggestion of Hultsch to take into account the Hipparchian periods for the Moon and the Sun was also not helpful in finding satisfactory solution (de Callatay, 1996).

In our paper we apply mathematical approach in order to find the original value of Proclus' Perfect Year. Our approach is based on the mathematical properties of the candidate numbers and identification of their factors with periods with known astronomical and/or metaphysical meaning.

2. METHODS

Our attempt to obtain the exact number for Proclus' Perfect Year and to identify the periods from which it was computed is based on careful examination of the properties of all 162 candidate numbers.

In order to find the missing digits of the Proclus' number we apply two reasonable assumptions formulated below:

Assumption 1. The number should include as factors some of the Babylonian and Egyptian planetary periods used in Neugebauer's reconstruction, or at least some of their prime factors.

Assumption 2. The largest prime factor of the number should be of same (or at least comparable) scale as the longest planetary period known to the ancients – the "greatest return" of Venus.

In addition, we must take into account that in the computations of the Perfect Year the accuracy was not of first importance for the ancients. One must consider as well that Proclus attributed to the perfect year not only scientific but also metaphysical properties, thus relating it to the philosophical concepts of eternity and totality (de Callatay, 1996).

For the prime factor decompositions of various candidate numbers $N(x,y)$ we use one of the prime factor calculators available in internet (Kourbatov, 1999), which allows decomposition of numbers containing up to 20 digits.

3. RESULTS

Let $P(x,y)$ be the highest prime factor of $N(x,y)$. On Figure 1 decimal logarithms of $P(x,y)$ are plotted versus x. The blue points correspond to $y = 9$ and the red points to $y = 70$, respectively. One can see that, although all of the prime factors are much higher than the largest of the "greatest returns" (1151 years), there is one number, namely $P(11{,}70) = 16\ 427$, which is much smaller than all other $P(x,y)$. The second smallest is $P(44{,}70) = 118\ 037$. It exceeds $P(11{,}70)$ more than 5 times.

If we look into prime factor decomposition of $P(11{,}70)$ we find it to be as follows:

$$N(11{,}70) = 2\ \times\ 5\ \times\ 19\ \times 487\ \times \\ \times 14\ 503\ \times 15\ 619\ \times\ 16\ 427.$$

This decomposition includes both a prime factor of the Sothic period (1461/3=487 years) and the 19-year Metonic cycle which was well known in the times of Proclus (Neugebauer, 1975). The properties of other prime factors, as we shall see below, make

N(11,70) an unique candidate for the true value of Proclus' Perfect Year.

When examining the ratios of the prime factors to the 1151-year period of Venus, we find following remarkable result:

$$14\,503 = 12.6 \times 1\,151 + 0.4,$$

or, after multiplying both sides of the above by 10:

$$145\,030 = 126 \times 1\,151 + 4.$$

In this way we find that 145 030 years is a product of Venus' "greatest return" and the 126-year period of Mars, known by ancient Babylonian astronomers, with a correction of 4 years.

One possibility worth to consider here is that the 4-year correction was introduced due to precession of the equinoxes. Despite precession was not accepted by Proclus (Siorvanes, 1996), it could be accepted by Sosigenes to whom Proclus attributed the computation.

A correction of 4 years over a time interval of approximately 145 000 years corresponds to precession of 1° per 100.7 years. This rate is very close to 1° per 100 years, computed by Hipparchus (Neugebauer, 1975) , and it is far away from much more accurate value of 1° per 72 years.

Similar number, 14 504, appears as time interval in years from Osiris to Amasis, derived from the Turin Royal Canon, including its mythological colimns (Palmer, 1861). This may be also considered as a confirmation of the significance of a period of approximate length 14 503 years for the ancient astronomers.

Another prime factor of $N(11{,}70)$, namely 15 619, is easily recognizable as 15 620-1, where 15 620 is the least common multiple of the smallest pair of amicable numbers $\{220, 284\}$. A pair of amicable numbers a and b by definition satisfies the equations

$$\sigma(a) - a = b$$

and

$$\sigma(b) - b = a,$$

where σ is the sum-of-divisors function.

The pair $\{220, 284\}$ is the earliest known pair of amicable numbers and the only one known by the time of Proclus. The Pythagoreans credited this pair with mystical properties.

One of the numbers in this pair has obvious astronomical meaning in our context. The "greatest return" of Mars is 284 years, and $284/4 = 71$ years is a Goal-Period of Jupiter (Hunger and Pingree, 1999).

The 220-year period, on the contrary, does not seem to be of any astronomical significance for the ancients. However, including 220 together with 284 in the computation could bear metaphysical significance in terms of reaching totality and perfection by including the other number of the amicable pair.

Another prime factor 16 427 is equal to 62 "greatest returns" of Saturn, with 3-year correction. The latter is too big to be explained by precession. Even if we take into account that the period was known by the ancients to be 5 or 6 days shorter than 256 years, this would contribute to a correction of 1 year only.

It is worth to mention also that 16427 years approximately equal 6 000 000 days. The equality becomes exact if we consider a year of 365.2523 days.

Regarding Mercury, none of the commonly used in ancient times periods of this planet, neither the "greatest return" of 480 years, neither the Goal-Year period of 46 years, could be unambiguously related to any prime factor of $N(11{,}70)$ or to any product of them. The closest approximation found is

$$14\,503\ 16\,427 = 496\,335 \times 480 + 19.$$

This correction, however, is too large. It equals about 4% of the planetary period itself. Indeed, fur such long time intervals precession can give even larger corrections, and one can consider the equation

$$14\,503 \times 16\,427 = (496\,335 - u) \times 480 - \\ -(19 + 480u).$$

Here u is unknown integer and the correction is

$$c = 19 + 480u.$$

Then, if the rate of precession in radians per year is p, in order to find u we have to solve the following equation:

$$14\,503 \times 16\,427 \times p = 2\pi(19 + 480u).$$

It is easy to see, however, that by choosing very similar values of p, one can obtain large number of different solutions to this equation.

The problem is additionally more complicated by the possibility that other, non-precessional corrections have been introduced.

4. DISCUSSION AND CONCLUDING REMARKS

In our attempt to reconstruct the Perfect Year of Proclus and its computation algorithm we considered the description of the computation given by Proclus. Both scientific and metaphysical meanings of the Perfect Year are taken into account. Appearance of the earliest known pair of amicable numbers is in good agreement with our approach.

In our partial reconstruction we have been able to identify periods corresponding to all planets, with the exception of Mercury. Our numbers, however, are not always exact, as they include corrections. In some cases precession is enough to explain the correction, while in other cases the corrections are too large to be explained by precession only.

Generally, the situation when small corrections of unclear origin are introduced in computations of the Great Year, of which Proclus' Perfect Year can be considered to be a special case, are not unique. Another, well known example are the never reconciled values 12 953 and 12 954 mentioned by different ancient authors for the Great Year (Griffin, 1979). Similarly to our case, 12 953 is a prime number, while 12954 is rich in divisors, having four prime factors:

$$12954 = 2 \times 3 \times 17 \times 127.$$

Although the reconstruction proposed is not perfect, all other candidate numbers $N(x,y)$ are far less promising than $N(11,70)$.

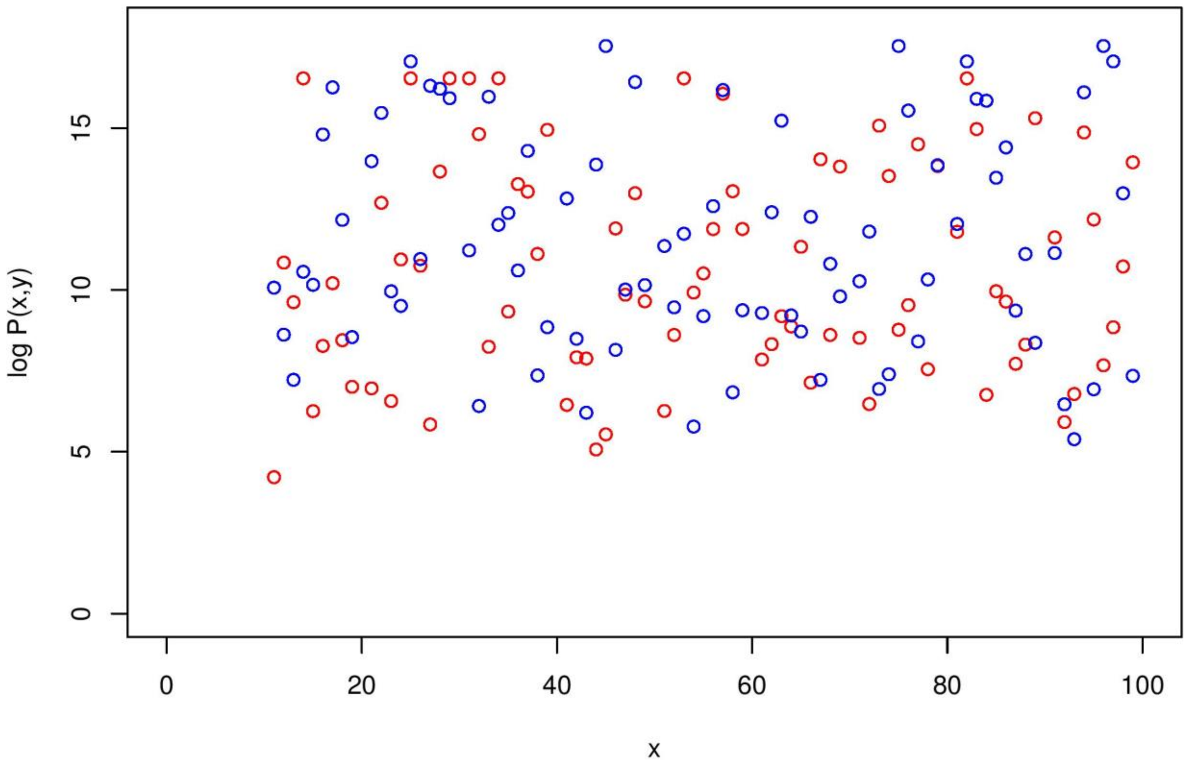

Figure 1. Decimal logarithms of $P(x,y)$ are plotted vs. x. The point corresponding to $(x,y) = (11,70)$ in in the left down corner is clearly separated from the other points.

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