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Research Article

Geometric Design Rules of Anatolian Roman Aqueducts

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ABSTRACT

Romans introduced many different building types in history, such as aqueducts, baths, and bridges. The bridges and arcades of the aqueducts are a few notable examples of Roman structures. Their design ensures their endurance as well as their attractive appearance. The sources of the geometrical design of Roman Aqueducts in Anatolia have yet to be deciphered. The purpose of the study is to determine the geometric design rules of Roman Aqueduct bridges and arcades located in Anatolia since most of them are in danger of loss and are not documented. There are no tracings of the top level or ground level of some aqueducts. The method of the study includes three phases. Firstly, the representative facade drawings of Roman Aqueducts were documented and drawn; secondly, a geometrical analysis using primary geometric forms such as circles and equilateral triangles; grids with Pythagorean triangles and golden ratio were made. Finally, analysis results were filtered to provide geometric structural schemes of the aqueducts. Consequantly, the usage of equilateral and Pythagorean triangles draws attention as much as the usage of circles. These triangles determine both the form of the arches and the facade organizations. Also, the grid system was used to determine some levels, such as cornices, springing lines, and the top of the arches. Thus, the golden ratio helped find the aqueduct's horizontal levels, especially the upper levels. The geometric schemes of the aqueducts are determined using all gathered data. Geometric schemes propose data about the aqueducts' design principles and for missing parts of the aqueducts.

Keywords: Roman Aqueducts, Restitution, Golden Ratio, Pythagorean Triangles.

INTRODUCTION

Romans are known for their advanced engineering achievements, such as bridges, aqueducts, roads, and other buildings. The idea of bringing water to the cities emerged with qanat buildings in the east. However, this idea was developed by the Romans with advanced geometric relations and engineering technology. The Romans found the arch, vault, and dome systems, even though they kept using columns or pillars as the foundation for figuring out the design of the Greek architectural system (Schumacher, Thiersch, Bühlmann, & Wagner, 1926). The bridges and arcades of the aqueducts are a few notable examples of Roman structures made of repeated arches. Their geometric design ensures their endurance as well as their attractive appearance. In order to determine the primary design decisions for the ancient aqueducts, it is crucial to analyse the geometric elements based on these constructions.

There are two different approaches determined to the design and construction of buildings or structures: the usage of modular systems using Pythagorean triangles and the usage of circles, squares, and equilateral triangles (Buchwald, 1992; Oikonomou, 2021). Triangle tracing can be used to determine the historic structures' rectangular perimeter and key structural elements. Using the basic right-angle triangle 3-4-5, the Pythagorean Theorem was demonstrated in numerous historic buildings. For tracing, additional triangles like 5-12-13, 8-15-17, 12-35-37, and 20-21-29 are especially useful (Oikonomou, 2021). Additionally, various geometric principles were examined, such as the golden section (ratio represented by the equation a/b= (a+b)/a) and dynamic rectangles connected to square roots (e.g., √2, √3, √5, etc.) (Dragović, Čučaković, Bogdanović, Pejić, & Srećković, 2019; Štambuk, 2002).

Three distinct topics in the literature were the subject of the geometrical analysis: architectural ornamentation and motifs, structural systems and elements, and plan and facade morphology. Proportioning systems based on dominating geometric shapes (triangle, square, and circle) are analyzed in plans, sections, and facades (Oikonomou, 2009, 2011, 2012, 2018, 2021; Bork, 2014; Dragovic, et al., 2019; Dragović, Čučaković, Bogdanović, Čičević, & Trifunović, 2020). In the geometric analysis of structural elements, research has been done on the contribution of geometric relations to the formation of all structural systems (Fuentes & Huerta, 2010), while there are also studies focused on the geometrical reproduction methods of the ornaments and motifs in architectural elements (B. Hajebi & P. Hajebi, 2021; Dabbour, 2012; Girón Sierra, 2022). There are studies focused on the geometrical rules of Greek and Roman Architecture (Heath, 1981; Jacobson, 1986; Wilson Jones, 2000; Duvernoy, 2021).

The principle of similarity, which was used especially in the facades of buildings in Greek architecture, was also used extensively in facade construction in Roman architecture (Schumacher et al., 1926). Schumacher et al. (1926) determined that the height and width ratio between the column axes and moldings on the facades of the Palazzo Pitti, Palazzo Strozzi in Florence, the buildings of Palladio were the same as the width and height ratio between the arch springing lines and the pillars. Circle and double squares inside the circle are the fundamental geometric features that establish the ground floor plans and sections, according to Dudley's (2002) analysis studies on the Pantheon. According to Fletcher's (2019) demonstration, the 3-4-5 triangle approach is used to trace the portico. In the analyses of the Pantheon Temple by Oikonomou (2021), 3-4-5 triangles in the plan and equilateral triangles in the section were determined. Despite not doing geometric analysis, Aicher (1995) and Hodge (2005) developed a classification system using the height and number of floors of the Roman aqueducts as well as their measurements.

Roman aqueducts are hardly the subject of specific geometric analytic research, however Roman building facades with features like arches and piers have been examined. Thus, the sources of the geometrical design of Roman Aqueducts in Anatolia have not been deciphered. The study aims to determine the geometric design rules of Roman Aqueduct bridges and arcades in Anatolia since most are in danger of loss and are not documented. There are no tracings of the top level and ground level of some aqueducts. Thus, a dataset for the restitution phases of the aqueducts aims to be gathered.

ROMAN AQUEDUCTS

Although the Romans are known for their vast and popular aqueduct constructions and waterworks, they were not the first ancient people who constructed this type of water channel and buildings. The Assyrians developed and widely used a tunneling technique called ‘qanat’ that is still used today in the Iranian Plateau (Ajam, 2003). Qanat buildings are a reliable water supply for living activities and irrigation in hot, arid, and semi-arid climates (Wulff, 1968; Ajam, 2003; Aicher, 1995).

In the Greek world, numerous towns had waterways before Roman occupation. The Greeks were the first to construct a waterway (Adam, 2005). The tunnel of Eupalinos is an essential Greek example of the water supply of Samos (Hodge, 2005). The Etruscans had also developed water channels for land drainage and water supply, called cuniculus. These water channels were almost similar to Qanat buildings. They are composed of tunnels through the soft volcanic rock and vertical shafts used for construction (Hodge, 2005).

The earliest waterway of Rome was composed of underground channels similar to Etruscan (4th century BC) techniques. Then, the construction of waterways became quite common throughout Italy as Appia, Anio Vetus, Marcia, Tepula, Julia, Claudia, and in towns from England to Africa. Most Roman Waterways are not in Rome; they were constructed in the provinces, such as the Valens Aqueduct in Turkey (Aicher, 1995; Adam, 2005; Hodge, 2005). Bridges and arcades are the first things that come to mind when considering waterways. Still, depending on landforms, a waterway comprises many parts, such as channels, pipes, cascades, settling tanks, arches, bridges, siphons, etc. Therefore, waterway building must include detailed planning, surveying, and construction.

Construction of a waterway required higher expertise to prepare the initial project drafts with the appropriate location and components. The first and most important thing for an engineer is to survey the source, route, and delivery point and then plan. However, cities were founded before the development of waterways. Therefore, the route could be inappropriate for bringing water. Landforms between the source and distribution point were surveyed and appropriate constructions for transferring water must be determined (Hodge, 2005) (Figure 1).

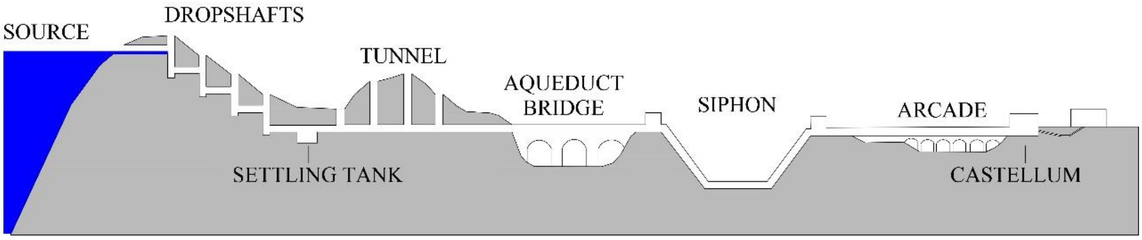

Figure 1. Components of a Roman Waterway (Drawn by the author)

Depending on land shapes, surface constructions vary in arcades, open channels, aqueduct bridges, and siphons. Arcades were composed of lines of arches. When the waterway entered the city, the water was tried to carry above the ground level for cleaning reasons. Therefore, arcades were mostly observed on plane grounds near the town. Alinda (Aydin), Phasalis (Antalya), Bergama (Izmir), and Iasos (Mugla) waterways have arcades (Öziş, 2015) (Figure 2).

Figure 2. Arcades in Alinda Ancient City, Aydin

Bridges composed of lines of arches were constructed whenever there was a valley to be encountered to shorten the route of the waterway. Therefore, bridges are usually comparatively shorter than arcades and are constructed between hills (Hodge, 2005; Passchier & Scram, 2004). There are lots of aqueduct bridges in Turkey as Pollio, Efes (İzmir), Valens (İstanbul), Paradiso (Izmir), Lamas (Mersin), Olba (Mersin), and Kargi (Aydin), etc. (Figure 3).

Figure 3. Pollio Aqueduct Bridge, Aydin

The study examined 11 Roman aqueducts. Three of them are aqueduct bridges, and 8 of them are arcades. Six aqueducts are one-storied, 4 of them are two-storied, and one of them is three-storied. The aqueducts are constructed with different masonry techniques. Bridges are Pollio Aqueduct, Efes; Aydin, Kursunlugerme, Istanbul; Sultaniye Aqueduct, Bahcecikbogaz, Aydin. The arcades are Alinda Aqueduct, Cine, Tyna Aqueduct, Kemerhisar; Milas Aqueduct, Mugla; Pisidia Antiokheia Su Kemerleri Aqueduct, Isparta; Selinus Aqueduct, Bergama; Kirkgoz (Limyra) Aqueduct, Side; Oymapınar Aqueduct, Side and Nikaia, Iznik Aqueduct (Figure 4).

Figure 4. Location of the Case Study Aqueducts (Revised from Yandex Map)

METHODOLOGY

The facades of eight arcades and three bridges were examined in the study. The study is composed of three phases. Firstly, the representative facade drawings of early Roman Aqueducts in Anatolia were documented and drawn; secondly, a geometrical analysis by using a grid with special triangles and a golden ratio was made. At last, the data gathered from the analysis was filtered to provide geometric structural schemes for the restitution of the damaged aqueduct bridges.

In the first phase, the facade of the aqueducts was documented, and the photos of the facades were rectified and scaled. Then, the facades were redrawn in Autocad 2022. Second, the facades of the structures are analyzed by geometrical methods to define the geometrical constructions. For the geometrical analysis, different methods were used in literature as the use of circles, squares, equilateral triangles, modular systems in combination with Pythagorean triangles (Oikonomou, 2009, 2011, 2012, 2018, 2021), Štambuk's proportional canon, which employs two circles constrained by an equilateral triangle in a specific setting (Stambuk, 2002), Dynamic rectangles related to square roots (e.g., √2,√3,√5, etc.) (Dragovic et al., 2019, 2020) the golden section (ratio expressed by the equation a/b = (a + b)/a); and "Continuous division" related to the Fibonacci sequence of numbers translated to rectangles (Kappraff, 1999; Dabbour, 2012; Roldan, 2012). Analysis of aqueducts was concerned with three subheadings:

1. Determination of geometrical constructions such as circles, squares, octagons, and equilateral or isosceles triangles

2. Determination of module or grid systems with the application of the cubit or foot in combination with Pythagorean triangles (3-4-5; 5-12-13; 8-15-17; 12-35-37, and 20-21-29 triangles)

3. Determination of the golden ratio

The Roman metric system, which is based on the foot (29.6 cm), cubit (44.4 cm), gradus (step of 74 cm), and ulma (four feet), adopts the logic of the ancient Greek metric systems (Smith, 1851). In this study, a cubit (44.4 cm) was used for the grid system.

RESULTS AND DISCUSSION

Geometric constructions that were determined from the analysis are square, circle, equilateral triangle, octagon, etc. It is also a determined grid with 3-4-5; 5-12-13; 8-15-17; 12-35-37, and 20-21-29 triangles and golden sections. It has been determined whether the geometric elements contribute to the facade organization or the design of the arch form. The data obtained were classified according to these three methods, and the findings are presented in the table below. The equilateral triangle, grid, and 20-21-29 triangle were the most prevalent geometries (Figure 5). Numerous investigations have been conducted to determine the composition of Roman arches. In the Roman era, the arches were designed based on circles (Tunç, 1978; Chitham, 1985; Ward-Perkins, 2003; Adam, 2005).

	Basic Geometries			Grid with speacial triangles											Golden ratio	
	Circle	Equilateral tri.		Grid	3-4-5 tri.		5-12-13 tri.		8-15-17 tri.		12-35-37 tri.		20-21-29 tri.			
		Arch	Facade Orga		Arch	Facade Or.	Arch	Facade Or.	Arch	Facade	Arch	Facade	Arch	Facade	Arch	Facade Or.
Alinda Aq., Aydın	√	√	√	√					√		√			√	√	/
Pollio Aq., Efes, İzmir	√	√	√	/			√	√			√	√			√	
Tyna Aq., Kemerhisar, Niğde	√	√						√						√	/	√
Milas Aq., Muğla	√	√	√	√				√						√	√	√
Kurşunlugerme Aq., İstanbul	√	√	√	/	√	√	√	√						√		/
Pisidia Antiokheia Aq., Isparta	√	√		√		√							√		√	
Selinus Aq., Antalya	√	√	√	√	√									√	√	
Kırkgöz Aq., Side, Oymapınar	√		√	√			√				√			√		√
Oymapınar Side Aq., Manavgat	√	√	√	√		√	√								√	
Nikaia Aq.,İznik	√	√		√	√									√	√	
Sultaniye Aq., Aydın	√	√	√	√		√	√	√	√							√

Figure 5. Geometric Elements Determined from the Analysis

As a result of the analyses, equilateral and Pythagorean triangles draw attention as much as the usage of circles. These triangles determine both the form of the arches and the facade organizations. The vertical or horizontal distance between the arches, the height of the bridges or arcades, the size of the arches, and the distance between the center of the arches with their piers are designed based on triangles. The organization of the triangles presents differences depending on the number of stories of the aqueduct. Also, the grid system was used for determining some levels, such as cornices, springing lines, and the top of the arches. However, the data gathered from the grid system is not as stable as from special triangles. Thus, the golden ratio helped find the aqueduct's horizontal levels, especially the upper levels. The organization of the triangles presents differences depending on the number of stories of the aqueduct.

Determination of Equilateral Triangles and Circles

At the facades of one-storied aqueducts, the top point of the equilateral triangles was placed at the aqueduct's highest level, while the triangle's lower points were placed at either end of the piers. This triangle forms a tangent to the circle defining the arch at two points. Two symmetrical equilateral triangles determine the height of the arches in some one-storied aqueducts (Alinda, Pollio, Tyna Aqueducts). The top point of the equilateral triangles was placed at the arch's highest level, while the triangle's lower points were placed at either the middle of the piers at the facades of some one-storied aqueducts (Selinus and Nikaia Aqueducts) (Figure 6).

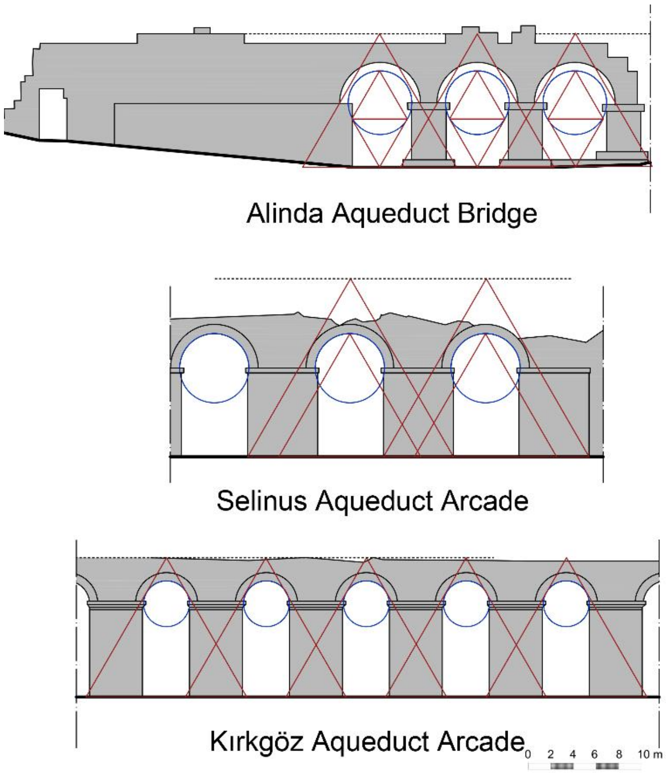

Figure 6. Basic Geometric Forms in One-storied Aqueducts

When the two or three-storied ones are examined, there are two types; arches of equal width positioned on top of each other (Oymapınar and Kursunlugerme Aqueducts) or small-sized arches placed at the upper levels, thus piers at the upper-level line up with the lower arches (Milas, Sultaniye and Pollio Aqueducts). At the ground level, the top point of the equilateral triangles was placed at the cornice level or top of the arches of the ground floor, while the triangle's lower points were placed at either end of the piers. The equilateral triangle rule, which determines where an aqueduct's or an arch's top is, is likewise followed in the upper elevations. Equilateral triangles of the same size establish the top limit of the building and the height of the arch in situations when the arches have the same width on both levels (Oymapınar Aqueduct) (Figure 7).

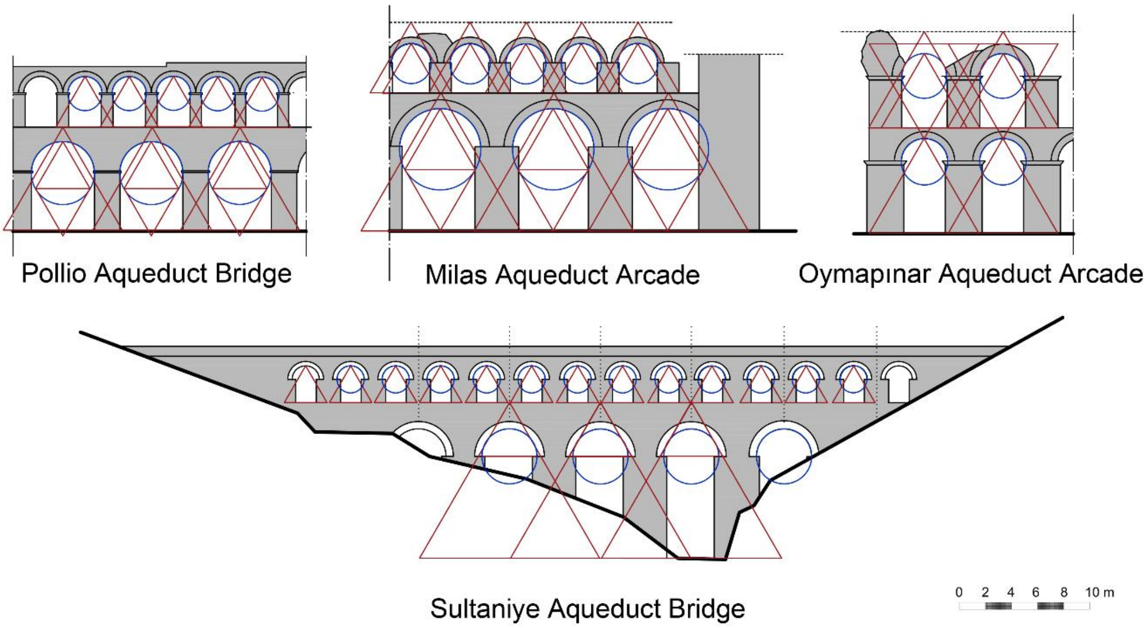

Figure 7. Basic Geometric Forms in Two-storied Aqueducts

Equilateral triangles appear to define the entire aqueduct facade like a pattern in the instance of the

Kursunlugerme aqueduct, which has three stories. The triangles starting from at the arch center of the ground level, continue up to the third level by connecting (Figure 8).

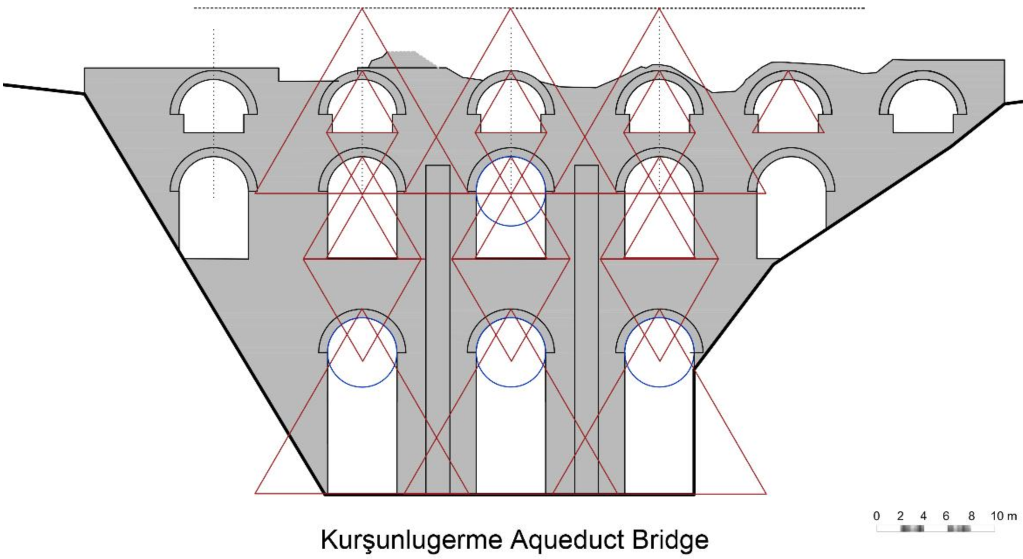

Figure 8. Basic Geometric Forms in a Three-storied Aqueduct

In the research of Oikonomou (2021), the equilateral triangle integrated with the perfect circle was found in the Pantheon section, although the equilateral triangles were not observed in the section and facades of the Roman arches in literature. Similar angles or parallel lines between the axes of the moldings, columns, and springing lines on the facade of Roman buildings were determined by Schumacher et al. (1926). Specific angles also highlight the special triangles found on the facade. Numerous parallel lines are identified at the facades of Kursunlugerme, Milas, Oymapınar, and Pollio Bridges (Figure 9).

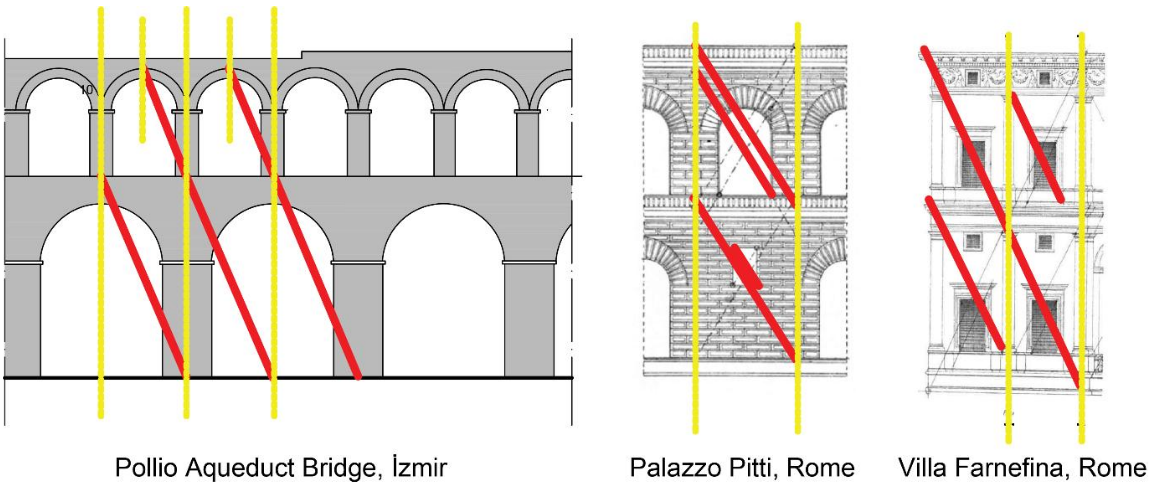

Figure 9. Parallel Lines Determined in Roman Buildings (Revised from Schumacher et al., 1926)

Determination of Grid System with Pythagorean Triangles

In the grid analyses with Pythagorean triangles such as 3-4-5; 5-12-13; 8-15-17; 12-35-37, and 20-21-29 triangles. The grid system was determined in all structures except for Tyna. In Kursunlugerme and Pollio Aqueduct, the grid presents only the top level of the arches and ground level; thus, it has not been applied consistently on the entire building facade. It has been found that each construction employs the grid and at least two different kinds of Pythagorean triangles.

The Pythagorean triangles that were most frequently used were 20-21-29. It was used to establish the facade layout's arch placements in 7 arches. In aqueducts, Pythagorean triangles are used to specify the heights, floor alignments, and tops of the arches. In Alinda, the central axis of the arch is aligned with the triangle of 8-15-17 from the end of the arch piers to the top of the structure. It also specifies the height of the building. The layout of the arches is determined by the triangle of 20-21-29 and 12-35-37 (Figure 10).

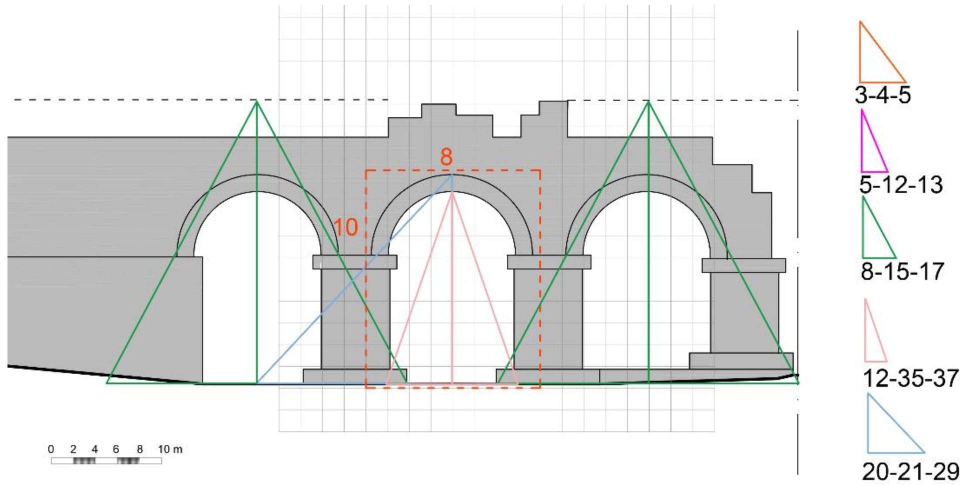

Figure 10. Grid with Pythagorean Triangles in Alinda Aqueduct

Triangles of 5-12-13 and 12-25-37 are utilized in Pollio aqueducts; the triangle of 5-12-13 has a pattern that identifies the middle axe of the structure as well as the piers and apex. The 5-12-13 triangle determined the positions of the arches on the lower and upper floors. The central axis of the arches is aligned with the triangle of 12-35-37 from ground level to the top of the arch. Also, the central axis of the upper arches is aligned from the piers' middle to the arch's top point. The arches' length-to-width ratio is 1.5 at the ground level and 1.4 at the upper level when the grids on which they sit are proportionate.

The Milas aqueduct follows a similar layout to the Pollio aqueduct; the triangle 5-12-13 designates the building's ground level, its apex, and the placements of arches at the ground and first level. The 20-21-25 triangle at the ground level provides the arch arrangement and apex, while the 3-4-5 triangle provides the upper floor's apex. The arches' length-to-width ratio is two at the ground level, and 1.5 at the upper level when the grids on which they sit are properly proportionate (Figure 11).

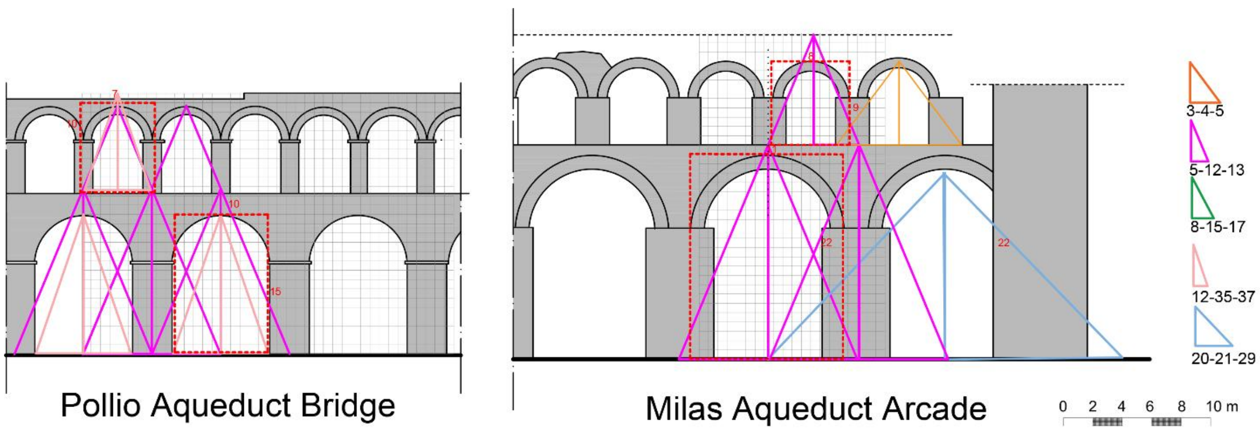

Figure 11. Grid with Pythagorean Triangles in Pollio and Milas Aqueducts

The 20-21-29 triangle provides the arch placements relative to the arch center in Tyna and Kirkgoz Aqueducts, whereas the 5-12-13 triangle specifies the arch apex in Tyna, arch center in Kıkrgöz. The arches' length-to-width ratio is 1.55 in Tyna, and 1.8 in Kirkgoz Aqueduct. The Kirkgoz Aqueduct's height is thought to be determined by the 12-35-37 triangle (Figure 12).

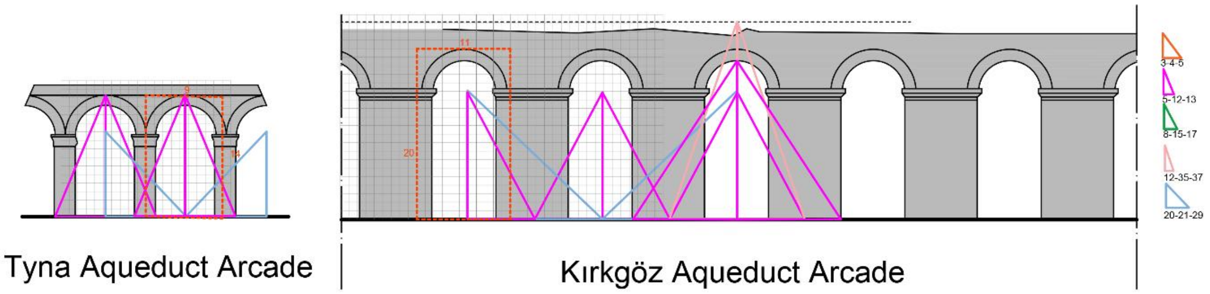

Figure 12. Grid with Pythagorean Triangles in Tyna and Kirkgoz Aqueducts

The arch patterns and heights in the Pisidia, Selinus, and Nikaia aqueducts are determined by the triangles 3-4-5 and 20-21-29. However, triangles define opposing roles in Pisidia. The arches' length-to-width ratio is 1 in the Yalvac Aqueduct, 1.5 in the Selinus Aqueduct, and 1.26 in the Nikaia Aqueduct. The Pisidia aqueduct's height is thought to be determined by the 12-35-37 triangle (Figure 13).

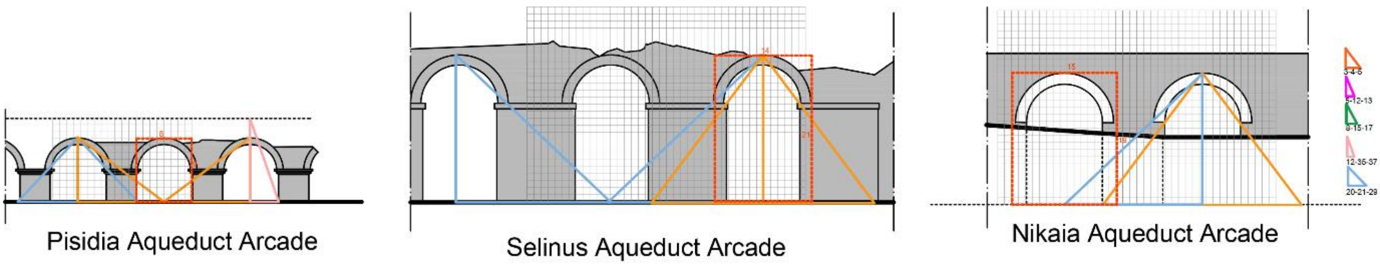

Figure 13. Grid with Pythagorean Triangles in Pisidia, Selinus, and Nikaia Aqueducts

OymapınarAqueduct has a different geometrical tracing compared to other two-storied ones since it has arches of equal width positioned on top of each other. 5-12-13 triangle completes the composition. The 3-4-5 triangle determines the lengths between the arches, whereas the 5-12-13 triangle determines the arches themselves. The arches' length-to-width ratio is 1.5 (Figure 14).

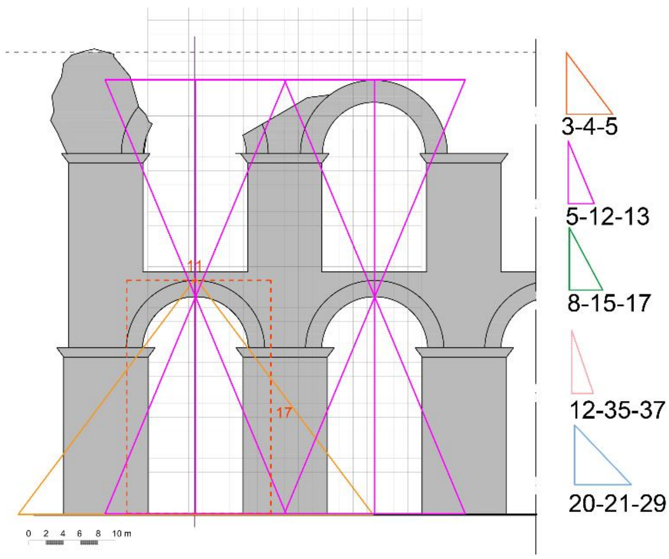

Figure 14. Grid with Pythagorean Triangles in Oymapinar Aqueduct

In Sultaniye Aqueduct, the ratio of the arch's width to its length at the ground level is 1.92, and the triangle that defines the shape of the arch is 5-12-13. The upper-level arch vertex is where the sidelines of the 5-12-13 triangle are. The arches on the upper level have a width-to-length ratio of 1.16, and the 8-15-17 triangle defines the shape of the arches. On both the ground floor and the upper level, the facade organization is determined using the 3-4-5 triangle (Figure 15).

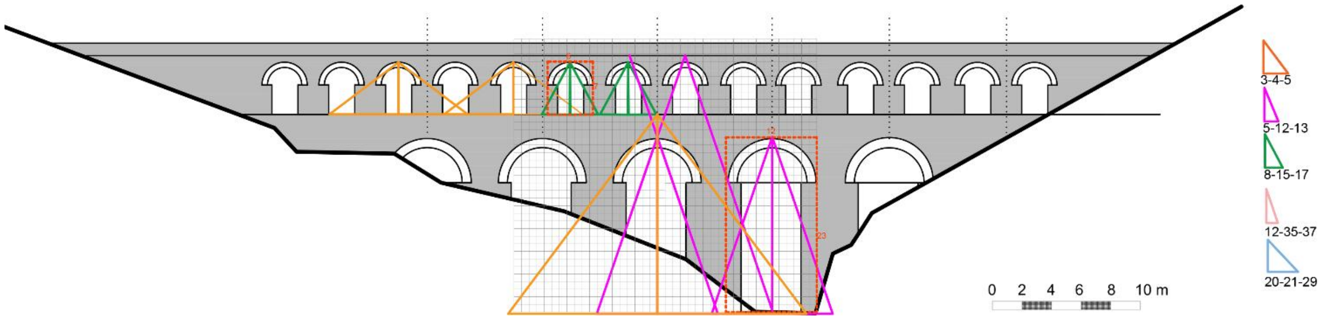

Figure 15. Grid with Pythagorean Triangles in Sultanhisar Aqueduct

In the three-story Kursunlugerme aqueduct, the 5-12-13 triangle is used frequently. While the triangle defines the ground floor arch form, it also establishes the floor level and placement of the arches on the upper level. While the 3-4-5 triangle represents the arch form at the third level, it determines the second floor's arch locations. The arches' length-to-width ratio is 2,14 at the ground level, 1.26 at the upper level, and 0.5 at the third level when the grids on which they sit are appropriately proportionate (Figure 16).

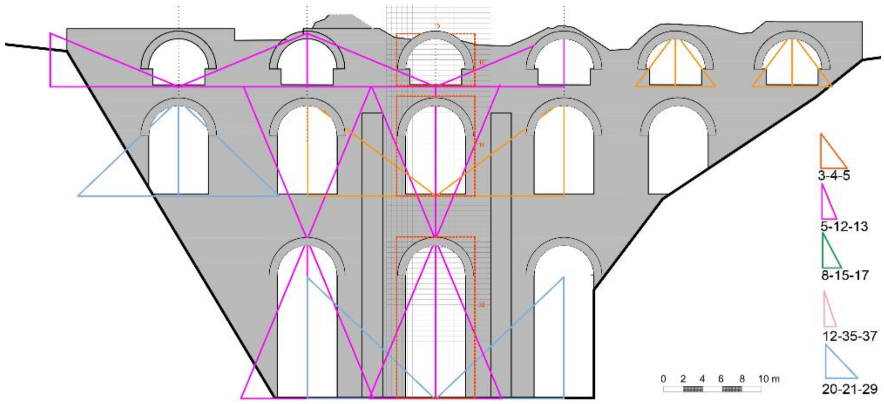

Figure 16. Grid with Pythagorean Triangles in Kursunlugerme Aqueduct

The similarity principle was not applied as frequently as it was in equilateral triangles, according to the analytical results of special triangles (Schumacher et al., 1926). The literature has shown that section end alignments are often determined by the usage of special triangles. Oikonomou (2021), determined the use of 3-4-5 triangles in the plan tracing of the portico of the Pantheon.

Determination of Golden Rectangle

The golden ratio is only sometimes utilized for determining horizontal lines for facade organization, such as springing and finishing lines. It is used for vertical pier alignment and the springing line, the extrados, and intrados apex horizontally in the arch form. The golden ratio covers both the side walls of the arch vertically and the springing line, the extrados, intrados, and the ground lines horizontally in the aqueducts of Alinda, Pollio, Milas, Kursunlugerme, Nikaia, Oymapınar, and Sultaniye. It does not contribute to the building's facade's geometric shape (Figure 17).

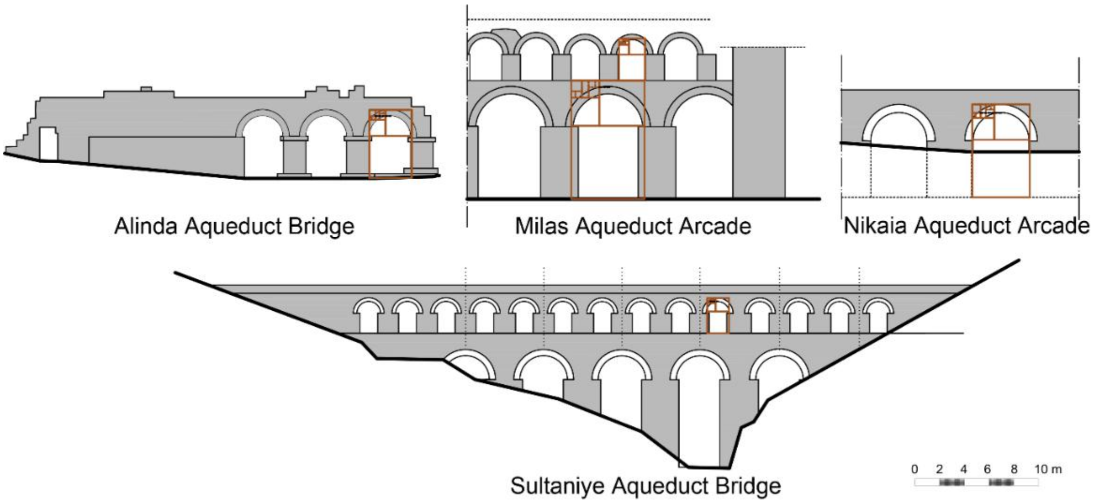

Figure 17. Golden Rectangles Determined in the Arches of the Aqueducts

The golden rectangle in Kursunlugerme and Pollio determines the placement of the cornice and the springing lines but is unable to construct the full facade (Figure 18).

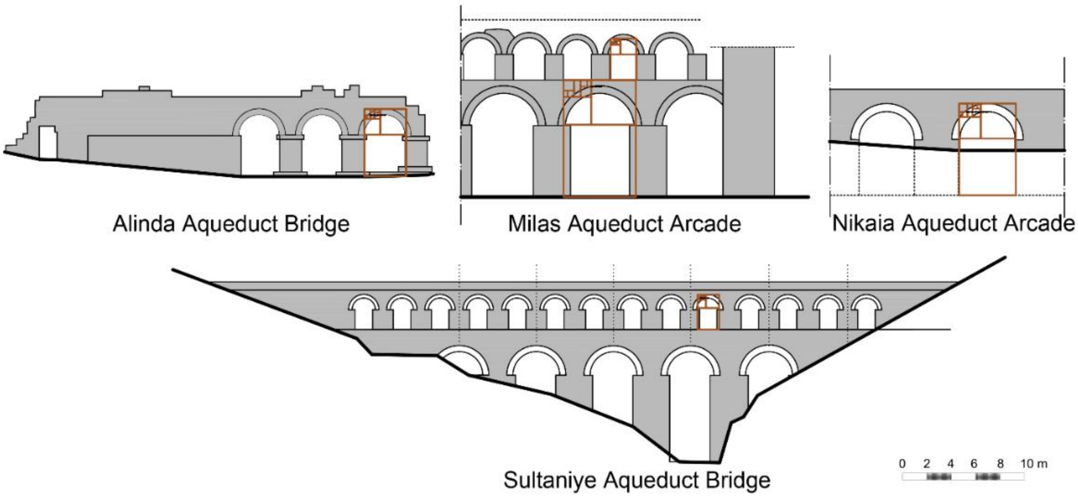

Figure 18. Golden Rectangles Determined in the Arches of the Aqueducts

The golden rectangle in Kursunlugerme and Pollio determines the placement of the cornice and the springing lines but is unable to construct the full facade (Figure 19).

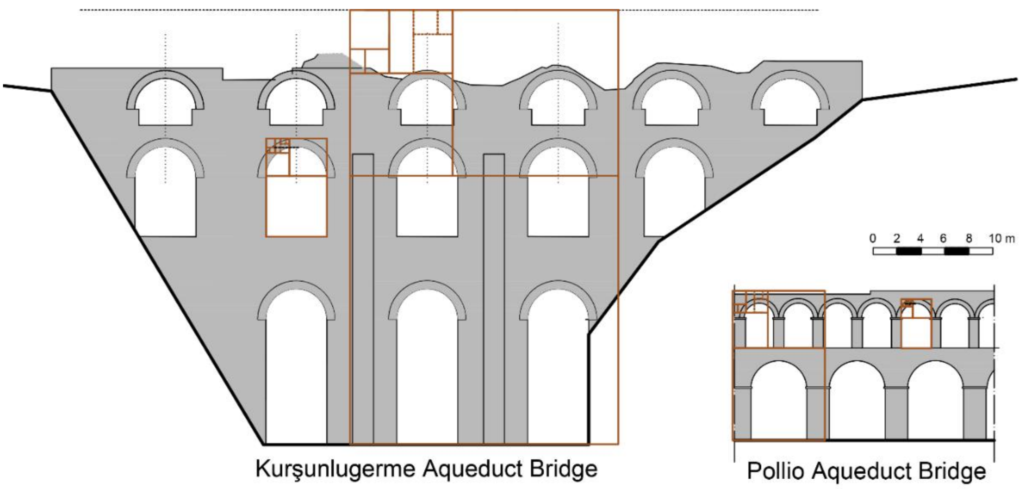

Figure 19. Golden Rectangles Determined in Kursunlugerme and Pollio Aqueducts

In Tyna aqueducts, the ground line, springing, cornice, and finishing lines of the facade are all aligned by a golden rectangle, while the arches are defined by a small-sized golden rectangle (Figure 20).

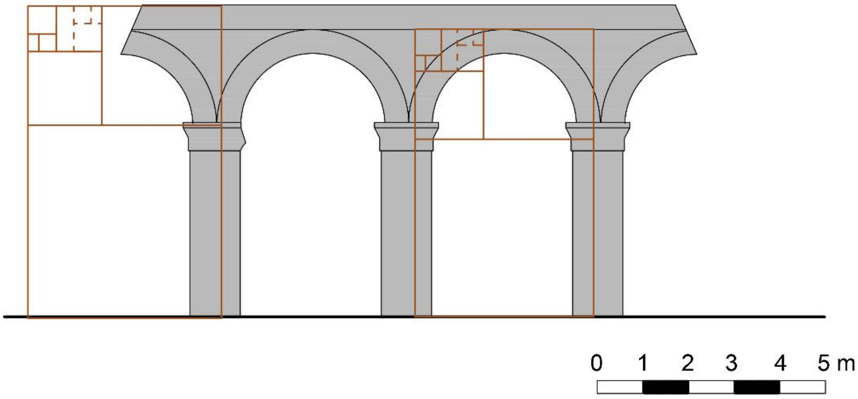

Figure 20. Golden Rectangles Determined in Tyna Aqueduct

A golden rectangle that defines the arch form and facade organization at the same time in the single-story Selinus and Kirkgoz aqueducts (Figure 21).

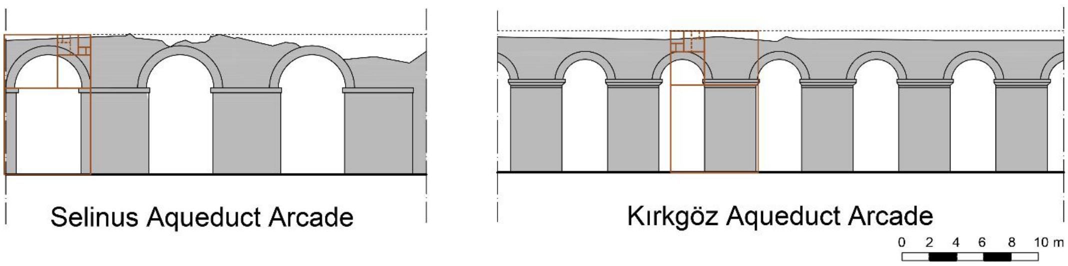

Figure 21. Golden Rectangles Determined in Selinus and Kirkgoz Aqueducts

THE GEOMETRIC STRUCTURAL SCHEMES PROPOSAL

The information gained from the analyses allowed for defining the geometric patterns of the aqueduct constructions. Diagrams specify the facade organization and arch form for single and two-storied aqueducts. The primary geometric components determined in the analysis were selected to serve as the schemes' framework. Since there aren't many examples of three stories of aqueducts, geometrical patterns are suggested for one and two stories.

There are two primary geometric schemes for single-story aqueducts. The circles in these designs determined the shape of the arch. The arch height and diameter were established by symmetrical equilateral triangles in Diagram 1 and by a single equilateral triangle in Diagram 2. The sequences of the arches are found using both equilateral triangles and Pythagorean triangles. The information from the triangles coincides with one another. The Pythagorean triangles that determine the arch configuration in aqueducts with width-to-height ratios under 1.5 are 3-4-5 or 20-21-29 triangles. The order of the arches is determined by the triangles 5-12-13, 8-15-17, and 12-35-37 if the width-to-height ratio of an arch is greater than 1.5. The golden rectangle defines the springing line, ground line, arch intrados, or extrados alignment. Since the golden rectangle data is the standard in determining the arch form, the data from the grid system was not used in the schemes. In the 1st diagram, the equilateral triangle determines the aqueduct finishing line; in the 2nd diagram, the golden rectangle determines (Figure 22).

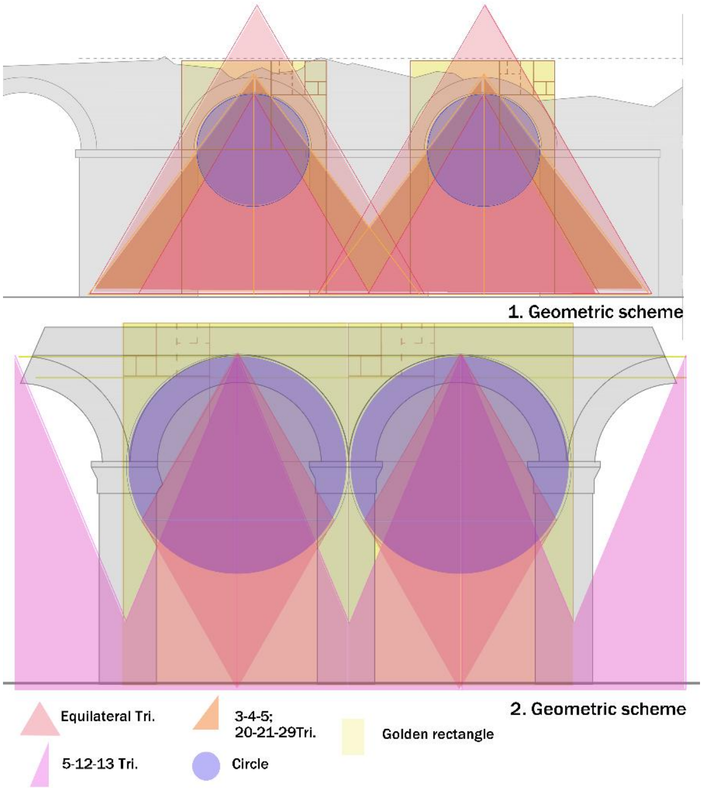

Figure 22. Geometric Schemes of One-storied Aqueducts

A one-storied aqueduct's geometrical forms are displayed in geometric diagrams. The second scheme illustrates the Tyna aqueduct's step-by-step facade formation (Figure 23).

1. Circles are drawn in step 1.

2. Inside the circle, equilateral triangles are drawn, along with their symmetries. The ground line is thus established.

3. The building end and springing lines are identified using the golden rectangle.

4. The 5-12-13 triangle determines where the arch piers are located.

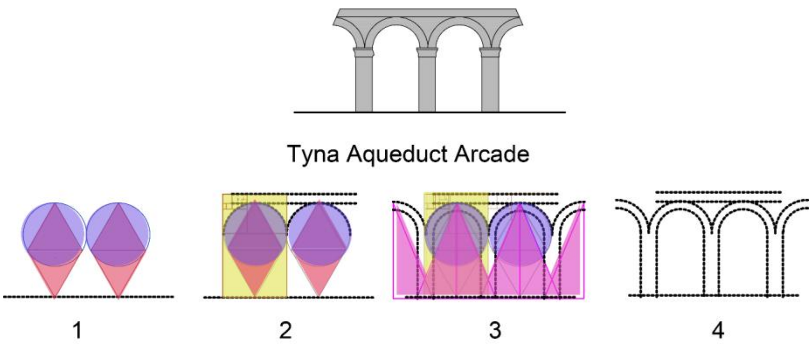

Figure 23. Tyna Aqueduct's Step-by-step Facade Formation

Two schemes for facade arrangement in two-storied aqueducts have been established depending on whether the arches will be placed on top of one another. Both equilateral and Pythagorean triangle-based schemes were identified. The triangle used to determine the arch arrangement in two-storied aqueducts is often the 5-12-13 triangle, and the ratio of the arch height to the width is typically more than 1.5. When this triangle is not utilized, the triangles 8-15-17 and 12-35-37 are. Equilateral triangles and the Pythagorean triangle are used to establish the order of the arches in two-storied aqueducts.

In scheme 3, small triangles on the higher level are situated at the apex of the large triangles on the ground level when the arches are not placed on top of one another. In scheme 4, the upper elevation arches' endpoint is defined by the symmetry of the 5-12-13 or equilateral triangles. The determination of springing lines and cornices in both designs has been made possible by the golden rectangle. Pythagoras and equilateral triangles were used to identify the structure's endpoint in the first diagram, while only equilateral triangles were used in the second (Figure 24).

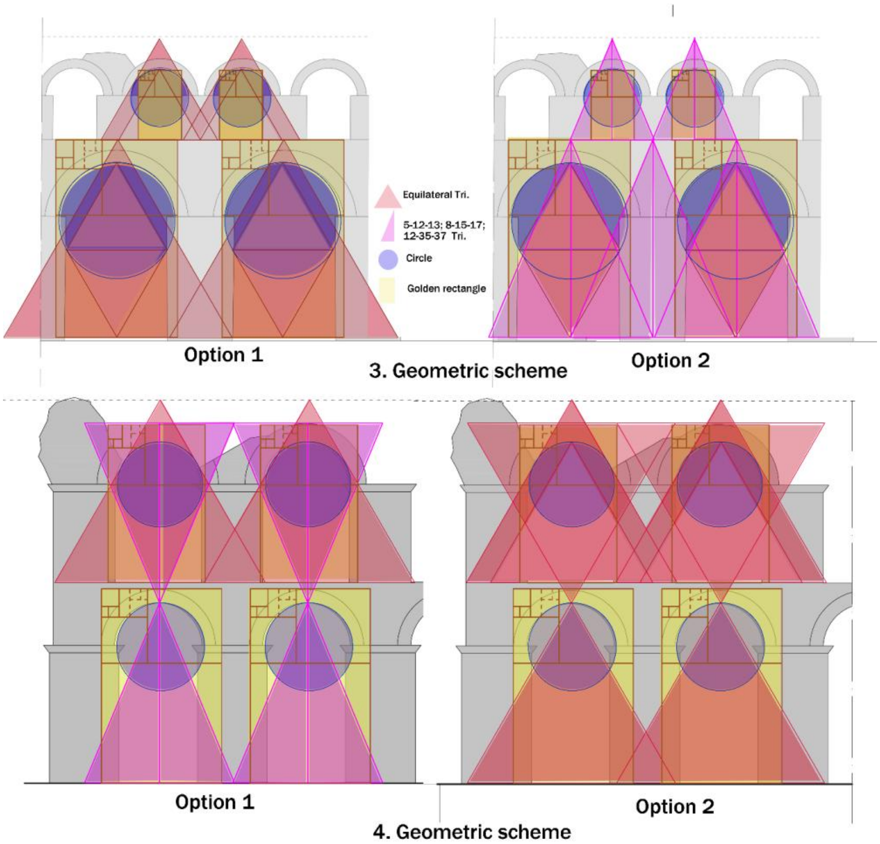

Figure 24. Geometric Schemes of Two-storied Aqueducts

A two-storied aqueduct's geometrical forms are displayed in geometric diagrams. The third scheme illustrates the Oymapınar aqueduct's step-by-step front formation.

1. Circles are drawn. Equilateral triangles created at the circle's vertex determine the ground level and cornices. The building's end and floor cornices are defined by the equilateral triangle drawn tangent to the circle at the higher level. Equilateral triangles' bottom corners represent where the piers should be aligned.

2. The golden rectangle establishes the alignment of the springing line and extrados.

3. It is apparent that the facade arrangement made possible by equilateral triangles and the golden rectangle is identical to that made possible by the 5-12-13 triangles (Figure 25).

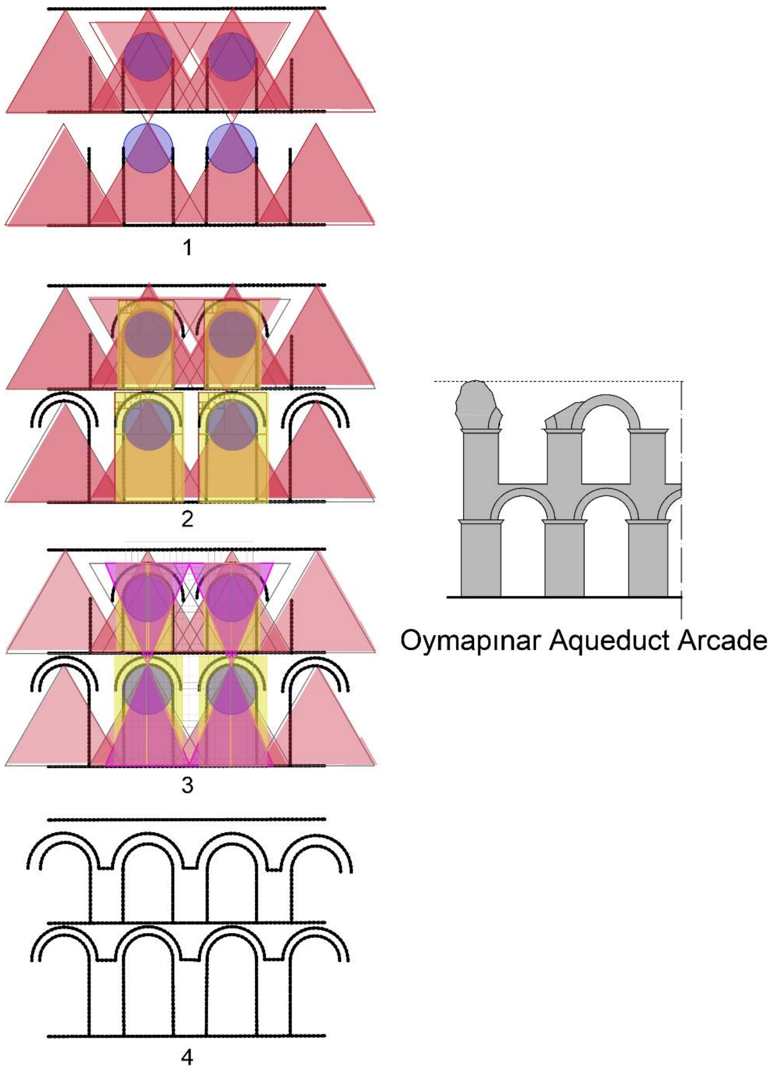

Figure 25. Geometric Schemes of Two-storied Aqueducts

CONCLUSION

Most of Anatolia's aqueduct bridges and arcades are in danger of loss and are not documented. They lost their integrity, and there were no documents to use for their restitution projects. The result of the geometrical analysis is an essential dataset for the restitution phases of the aqueduct, and it defines the fundamental design principles of Roman Aqueduct bridges and arcades in Anatolia. The importance of knowledge of geometry in construction is also deciphered.

Geometric forms can be interpreted as a square, circle, equilateral triangle, octagon, and grid system, combining Pythagorean triangles (3-4-5; 5-12-13; 8-15-17; 12-35-37, and 20-21-29 triangles). It was seen that the triangles were used to design the distance between the arches and the level between the ground and the top of the arches. The position of consecutive and upper and lower arches was developed based on The Pythagorean and equilateral triangles. The Pythagorean triangles that were most frequently used were 20-21-29. It established the facade layout's arch placements in 7 arches. In aqueducts, Pythagorean triangles are used to specify the heights, floor alignments, and tops of the arches. Also, the grid system was used for determining some levels, such as cornices, springing lines, top of the arches. However, the data gathered from the grid system is not as stable as the data collected from special triangles. Thus, the golden ratio helped find the aqueduct's horizontal levels, especially the upper levels. The arches ' top and cornice levels, intrados, and extrados were designed based on the golden ratio. As well as the structure proportions, the golden ratio presented the proportions of arches individually. Pythagorean triangles of all types and the golden ratio were also taken into consideration in addition to fundamental geometric shapes.

The geometric schemes of the aqueducts may be determined using all information gathered from the analysis. 4 schemes were determined for one-storied and two-storied aqueducts. It is seen that the circles, triangles, and golden rectangles identify the facade design of Roman Aqueducts. Through their mutual completion, they create a facade organization. The circles in these designs determined the shape of the arch. Symmetrical triangles and a single equilateral triangle established the arch height and diameter. The arches' sequences are found using equilateral triangles and Pythagorean triangles. The information from the triangles coincides with one another. The Pythagorean triangles that determine the arch configuration in aqueducts with width-to-height ratios under 1.5 are 3-4-5 or 20-21-29 triangles. The order of the arches is determined by the triangles 5-12-13, 8-15-17, and 12-35-37 if the width-to-height ratio of an arch is higher than 1.5. The golden rectangle determines mostly horizontal alignments at the facade, also equilateral triangles or golden rectangles determine the aqueduct finishing line.

Geometric schemes provide data both about the design principles of the aqueducts and also for missing parts of the aqueducts. Especially in Anatolia, the aqueduct bridges have lost their finishing and ground lines, and there is not a trace of it in the structures. The geometric schemes can be used as a layout to complete missing parts in the restitution studies of the aqueducts. The actual height of the bridges and arches can be deciphered.

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