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CONICAL SUNDIALS FROM THE HELLENISTIC AND ROMAN PERIOD AT MUSEUMS OF ATHENS: NOVEL TIME MEASUREMENTS CONCERNING OPERATION AND ACCURACY

Evangelia Panou[1], Xenophon Moussas[2] & Panagiota Preka-Papadema[2]

[1]University of the Aegean, Dept. of Mediterranean Studies, Lab of Archaeometry, 1 Demokratias Str, Rhodes, Greece
[2]Department of Astrophysics-Astronomy and Mechanics, Faculty of Physics, National and Kapodistrian University of Athens, Panepistimiopolis, Zographos 157 84, Athens, Greece

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Corresponding author: Evangelia Panou (evipanou184@yahoo.gr)

ABSTRACT

The purpose of this work is to study fourteen ancient sundials from in-situ measurements and discuss the significance and importance of time measurements in antiquity. The time measuring marble devices have been found in Attica, Greece (at National Archaeological Museum, Piraeus Museum, Stoa of Attalos at the Athens Agora, Epigraphical Museum of Athens). These sundials date from the Hellenistic and Roman period. Two of them still have the original gnomons. Six out of fourteen are preserved in good condition; the others are fragments of the original sundials. Detailed in-situ measurements along their present-day hour lines and curves (solstices and equinoxes) are taken. The methodology followed is based on Gibbs' methodology for south-facing conical sundials with slightly modifications/alterations depending on the present-day grid of lines and curves of the sundials. Conclusions about their geographical latitude of operation and their accuracy construction are drawn.

KEYWORDS: cultural heritage, conical sundial, time, astronomy, geographical latitude, sundials, gnomon, Stoa of Attalos, Archaeological Museum of Piraeus, Ancient Agora of Athens, Epigraphical Museum of Athens

1. INTRODUCTION

Early man divided the year into smaller time periods and invented time systems for recording date. As civilization became more developed, the need arose to divide the day into indefinable time intervals (Lennox-Boyed, 2005). This need led the sky observation to be considered of vital importance in ancient times.

Concerning Greek antiquity, astronomers and mathematicians of the Classical Greece (479-323 B.C.) and the Hellenistic period (323-31 B.C.) contributed to the foundations of the science of astronomy by tracking the movements of the celestial bodies and devising methods and instruments (portable or immovable) to project their orbits on the Earth's surface (Seiradakis & Edmunds, 2018; Hannah, 2015; Spyridis et al., 2019; Jones 2017).

Anaximander of Miletus was the first to construct a time-telling instrument (erected at Sparta, Greece), the gnomon, for the identification of solstices, equinoxes, time spans and hours (Thibodeau, 2017).

As the mathematic knowledge advanced, the first sundials for measuring time during the sunny days were devised. They were widely used in the regions around the Mediterranean Sea due to the sunny weather. Sundials were usually placed in open air spaces such as in the central square of a town or in yards of manor houses. This signifies that people were familiar with the notion of time. Archaeoastronomy today aids the investigation of the capabilities of ancient people, yet supply us with valuable measurements on astronomical parameters (Liritzis et al., 2020).

The sundial is a marble construction consisting of a dial plate (flat or concave surface), which bears carved lines and curves, and a gnomon (protruded stick that casts shadow on the plate) (Fig. 1). When the sundial is exposed to the Sun light a shadow is created on the dial plate due to the gnomon. Both the shadow position on the dial plate and the length of the shadow vary depending on the daily hour and the season.

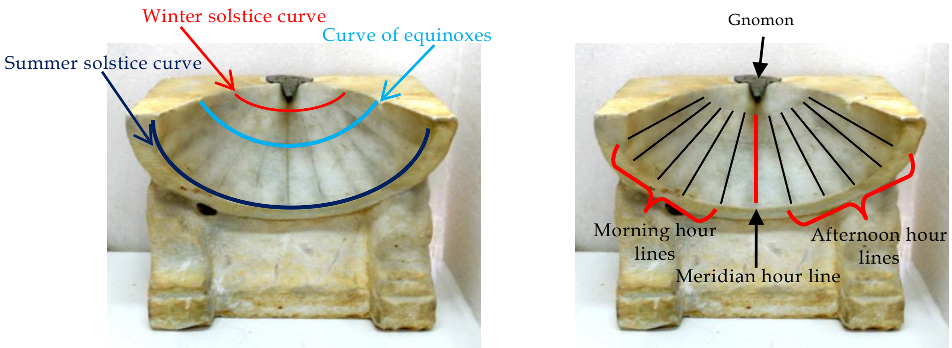

Figure 1. Example of a south-facing conical sundial
(1a) The winter solstice curve, the curve of equinoxes and the summer solstice curve are marked.
(1b) The morning hour lines, the afternoon hour lines, the meridian hour line and the gnomon are marked. The sundial (index number No 3158) displayed is kept at the Archaeological Museum of Athens, Greece
©Panou (2016a)

There are various types of ancient sundials. The most common sundials from Graeco-Roman antiquity are of conical type. They have a face which is part of the surface of a cone with its axis (KN) parallel to the axis of the earth (see Fig. 2). Its face is bounded at the top by a plane parallel to the horizon and in front by a plane parallel to equator. Such a conical sundial with three day curves and eleven hour lines (following the curvature of its dial plate) carved on its dial plate is depicted in Fig. 1. This sundial is also a typical example of south-facing conical sundials. This implies that its dial face is part of a right circular cone1 with vertex (angle ρ) above the horizon (the parallel plane that the dial is bounded at the top) (Fig. 2).

Starting from the base of the gnomon the first day curve corresponds to the winter solstice curve; the second one to the curve of equinoxes; the latter to the summer solstice curve (Fig.1a). Starting from the left-hand side of the construction the first hour lines indicate sunrise and morning hours. The meridian hour lying under the gnomon indicates solar noon and the other hour lines are for afternoon hours and sunset (Fig. 1b). Using conical sundials, the daily hour and the season of the year can be determined by the relative position of the shadow and its length among the lines (known as hour lines) and among the curves (known as day curves indicating the equinoxes and solstices) respectively.

[1] Right cone: cone that has its apex aligned directly above the center of its base.

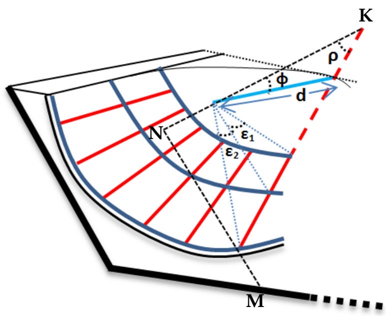

Figure 2. Schematic presentation of a conical sundial. In the figure, the angle (ρ) formed between the cone-axis (dark dashed line) and the generatrix (red dashed line), the geographical latitude (φ) of operation, the length (d) of the gnomon (bold blue line) and the angles of the obliquity of the ecliptic ($\varepsilon_1=\varepsilon_2=\varepsilon$) are marked.

Sundials are unique per location; they only operate at specific geographical latitude. Thus, their manufacturing accuracy is of highly importance. The manufacturing accuracy of conical sundials can be determined by measuring three characteristic parameters (Fig. 2): a) the angle (ρ) formed between the cone-axis (coincide with the Earth's axis) and the generatrix (coincide with the meridian hour line), b) the geographical latitude (φ) of operation (formed by the cone-axis and the gnomon) and c) the length (d) of the gnomon (Gibbs, 1976; Panou 2016a). Moreover, the value of the obliquity of the ecliptic also indicates the manufacturing accuracy of a sundial and can be determined purely geometrically. The angle of the obliquity of the ecliptic (ε) is equal to the angles formed by two rays emanating from the peak of the gnomon and ending to the dial plate at the intersection points formed by the curves and the meridian line. The sundial is accurately constructed when the angle (ε_1) is equal to the angle (ε_2).

Table 1 lists the fourteen studied sundials. Among the fourteen only two had gnomons. It is the first time that six out of fourteen sundials have being studied thoroughly. The other eight out of fourteen sundials have already been studied. However, new detailed measurements are taken and a reassessment is made.

Table 1. The fourteen studied sundials.

No.	Sundial No.	Museum	Researchers	Arch. Reference
1	3158, g	NAMA	Panou; Gibbs	Schaldach
2	3156	NAMA	Panou; Gibbs; Rayet	Schaldach; Heydemann
3	3157	NAMA	Panou; Gibbs	Schaldach; Heydemann; Von Sybel
4	A 1870	SAAA	present	Schaldach; Gibbs
5	A 1769	SAAA	present	Schaldach; Gibbs
6	ST 148	SAAA	Panou; Gibbs	Schaldach
7	ST 209	SAAA	Panou; Gibbs	Schaldach
8	ST 147	SAAA	Panou; Gibbs	Schaldach
9	ST 780	SAAA	Panou; Gibbs	Schaldach
10	ST 589	SAAA	present	Schaldach; Gibbs
11	EM 9818	EMA	present	Schaldach; Gibbs
12	EM 2922	EMA	present	Schaldach; Gibbs; Graindor; Foucart
13	EM 2094	EMA	present	-
14	MP1131, g	MP	Panou; Páris; Gibbs	Schaldach 2006

NAMA: National Archaeological Museum of Athens; SAAA: Stoa of Attalos at Agora of Athens; EMA: Epigraphic Museum of Athens; MP: Museum of Piraeus; g: with gnomon

For first time here the value of the obliquity of the ecliptic has been calculated by experimental measurements thanks to the existing gnomon of the sundial with index number 3158. For the twelve sundials the value of the obliquity of the ecliptic cannot be calculated experimentally. Thus, for our calculations the value calculated by Eratosthenes ($\varepsilon=23.86^o$) during the 3rd century B.C. (Pappus, 1876; Jones, 2002) is used.

The objective here is to take accurate measurements (lengths and angles) on the fourteen sundials so as to check the operativity and accuracy of these ancient marble devices.

2. METHODOLOGY

The methodology followed for the calculation of the characteristic parameters of the sundials depends on their present-day grid of lines and curves. For methodology purposes the sundials are divided into four categories:

a) sundials with complete grid and gnomon (sundials with index number 3158 & MP 1131),

b) sundials with complete grid line and no gnomon (sundials with index numbers 3156, 3157, ST 147, ST 780 & EM 2094),

c) sundials with no complete grid (missing lines or curves) (sundials with index numbers EM 2922, ST 148 & EM 9818) and

d) fragments of sundials (fragments with index numbers A 1870, A 1769, ST 209 & ST 589).

The methodology used in our study and the mathematical types used, is based on Gibbs' methodology for south-facing conical sundials (Gibbs, 1976) with slightly modifications/ alterations (Panou, 2016a). It states that we modified Gibbs' mathematical types using the right triangle properties and the exterior angle theorem of triangles with respect to the shape of the fragments.

For our measurements the following points are considered:

- The geometrical characteristics of the constructions are measured and detailed measurements on dial plates are followed.
- The dimensions of the gnomon of the sundials (sundials with index numbers 3158 and MP 1131) are also measured.
- The value of the obliquity of the ecliptic is determined experimentally by measurements taken on sundials with gnomon.

Concerning the calculations:

- The geographical latitude of operation of the sundials is calculated.
- The angle between the axis and the generatrix is calculated.

The initial length of their gnomon is calculated. The difficulty arises here is that many sundials have not their initial shape; they are fragments. In these cases, the broken hour lines and curves are theoretically extrapolated using a ruler and a compass with respect to the geometrical characteristics/dimensions of the existing part of the dial plate. The measurement error is no greater than 0.1cm. The value of the obliquity of the ecliptic used for further calculations for sundials with no gnomon is ε=23.86°. Concerning the methodology followed:

a) For sundials with complete grid and gnomon the methodology is mainly based on Gibbs' methodology. The value of the obliquity of the ecliptic is measured experimentally (see also Fig. 2), so the characteristic parameters of the sundials are calculated based on its experimental values.
b) For sundials with complete grid and no gnomon the methodology is also based on Gibbs' methodology. The value of the obliquity used for our calculations is ε=23.86°.
c) For sundials with no complete grid (missing lines or curves) the methodology used is based on measurements of their geometric dimensions. The geographical latitude (φ) is measured directly; the angle (ρ) is calculated from measurements and trigonometric ratios and the length of the gnomon (d) is calculated using modified trigonometric formulas based on Gibbs' methodology. To be more specific the angles measured are (Fig. 3): i) the angle (φ) with vertex the intersection point of the horizontal plane parallel to the right or left-hand side of the base of the construction and the extrapolation of the plane that passes through the lower part of the dial plate which is equal to the geographical latitude, ii) the angle (b) with vertex the intersection point of the vertical plane that passes through the peak of the gnomon and the extrapolation of the meridian hour line. Moreover, trigonometric formulas are used for calculating the angle (ρ) and the length of the gnomon (d) (see Panou 2016a, pp. 299-301).
d) For sundial fragments the methodology used is a two-step methodology: i) depending on the shape and geometry of the fragment, extrapolation of its hour lines and curves are made, ii) Gibbs' methodology for south-facing conical sundials with respect to the geometrical characteristics of the fragments is used.

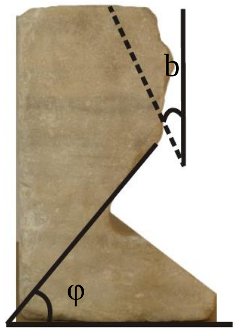

Figure 3. Schematic presentation of the angles (φ, b). ©Panou (2016a)

3. MEASUREMENTS

Measurements on sundials depend on whether or not the sundial has carved lines and curves on its dial plate. For sundials with complete grid or well-preserved dial plate measurements along the hour lines and curves are taken. For sundials with incomplete grid measurements on the external surface of the construction are taken. For sundials with index numbers No 3158 and MP 1131 the length of their gnomons is also measured by a ruler and a compass. Then, calculations followed according to the methodology previously mentioned. Below we first present the newly measured six sundials, and follows the eight reassessed ones.

3.1. New measurements

The sundials with index numbers A 1870 (Fig. 4), A 1769, ST 589, EM 2922, EM 9818 (Fig. 5), EM 2094 (Fig. 6) date from the Hellenistic or the Roman Period (see also Table 1) and have already been recorded (Fig. 4, 5, 6). However, their characteristic parameters (φ, ρ, d) have never been calculated.

Figure 4. Fragment of the conical sundial A 1870 (left) and view of its broken dial plate (right) ©Panou (2016a)

All sundials have carved hour lines on their dial plate except the sundials EM 2922 and EM 9818 that have only carved lines. Moreover, the sundials EM 2922, EM 9818 and EM 2094 bear ancient Greek engraved inscriptions.

Figure 5. Fragments of the conical sundials A 1769 (up and left), ST 589 (up and right), EM 2922 (down and left) and EM 9818 (down and right) ©Panou (2016a)

Figure 6. The conical sundial EM 2094 ©Panou (2016a)

The sundial EM 2094 (assemblage of three fragments) is the only sundial with obvious lines and curves on its dial plate (Fig. 6).

Table 2 lists the calculated values of the characteristic parameters (ρ, φ, d) of the fragments. For the sundials with carved hour lines and curves the values φ_1 and φ_2 of geographical latitude of operation are calculated by measurements taken between the winter solstice curve and the curve of equinoxes or/and by measurements taken between the summer solstice curve and the curve of equinoxes respectively. The values d_1 and d_2 refer to the initial gnomon length of the sundials and are calculated using φ_1 and φ_2 values respectively.

The sundials EM 2922 and EM 9818 have only carved lines (no curves). Thus, the geographical latitude of operation φ is measured experimentally and the calculation of the initial gnomon length of sundials follows.

3.2. Reassessed Measurements

The sundials with index numbers 3158, MP 1131 (Fig. 7), 3156, 3157, ST 780 (Fig. 8), ST 148, ST 209, ST 147 (Fig. 9) date from the Hellenistic period, the Roman period or late antiquity. The two first sundials still have gnomons. Despite the fact that these sundials have already been recorded and measured, reassessed measurements are taken and conclusions are drawn for the accuracy of their construction.

Figure 7. The conical sundials No 3158 (left) and MP 1131 (right) with gnomon ©Panou (2016a)

The sundials 3158, MP 1131, 3156, 3157, ST 780, ST 147 are conical sundials with complete carved grid on their dial plates (three day curves and eleven hour lines). The ST 148 and ST 209 are conical fragments with incomplete carved grid. Concerning the two sundials with gnomon, the gnomon of the sundial 3158 is horizontal and parallel to the imaginary plane passes through the upper part of the sundial while the gnomon of the sundial MP 1131 visibly slants.

Table 3 lists the calculated values of the obliquity of the ecliptic and of the characteristic parameters (ρ, φ, d) of the eight reassessed sundials. The values ε_1

and ε_2 of the obliquity of the ecliptic are measured experimentally (see also Fig. 2) and their values are used for the calculation of the characteristic parameters of the sundials with gnomon. The values φ_1 and φ_2 of geographical latitude of operation are calculated by measurements taken between the winter solstice curve and the curve of equinoxes or/and by measurements taken between the summer solstice curve and the curve of equinoxes respectively. The values d_1 and d_2 refer to the initial gnomon length of the sundials and are calculated using φ_1 and φ_2 values respectively. Table 3 also lists other researchers' results for the eight sundials.

Figure 8. The conical sundials No 3156 (up and left), No 3157 (up and right) and ST 780 (down) ©Panou (2016a)

Figure 9. The fragments of sundials ST 148 (up and left) and ST 209 (up and right). The slightly broken conical sundial ST 147 and the upper view of the base of its gnomon (down). ©Panou (2016a)

Table 2. Calculated values of the characteristic parameters (ρ, φ, d) of the newly measured six sundials

Index number of sundial	Angle between the axis and the generatrix ρ (degrees)	Geographical latitude φ_1 (degrees)	Geographical latitude φ_2 (degrees)	Geographical latitude φ (degrees)	Length of the gnomon d_1 (cm)	Length of the gnomon d_2 (cm)	Length of the gnomon d (cm)
A 1870 [d]	46.3 ± 2.2		40.7 ± 2.0			10.0 ± 0.3	
A 1769 [d]	46.2	37.1			7.3		
ST 589 [d]	53.5	37.3			13.6		
EM 2922 [c]	27.5 ± 1.0			37.8 ± 1.0			10.8 ± 0.2
EM 9818 [c]	33.2 ± 0.4			38.0			8.2 ± 0.1
EM 2094 [b]	7.6	43.7	57.9		24.5	20.0	

φ_1: calculated by measurements taken between the winter solstice curve and the curve of equinoxes; φ_2: calculated by measurements taken between the summer solstice curve and the curve of equinoxes; φ: measured experimentally; d_1, d_2, d: calculated using φ_1, φ_2, φ respectively;
[a], [b], [c], [d]: sundial's category according to methodology followed for their study (see paragraph ''2. *METHODOLOGY*'')

Table 3. Calculated values of the obliquity of the ecliptic and of the characteristic parameters (ρ, φ, d) of the eight reassessed sundials

Index number of sundial	Researchers	Obliquity of the ecliptic ε_1	Obliquity of the ecliptic ε_2	Theoretical value of the obliquity of the ecliptic ε	Angle between the axis and the generatrix ρ (degrees)	Geographical latitude φ_1 (degrees)	Geographical latitude φ_2 degrees)	Geographical latitude φ (degrees)	Length of the gnomon d_1 (cm)	Length of the gnomon d_2 (cm)	Length of the gnomon d (cm)
3158 [a]	Panou (2016a)	28.0	31.0		25.8			42.0			5.9*
	Gibbs (1976)			24.0	36.6	42.0	34.5	32.2			5.9*
MP 1131 [a]	Panou (2016a;b)	20.0	32.7		48.4 (using ε_1) 32.6 (using ε_2)	57.5	41.7				~4.0 (6.8*)
	Páris (1914)	24.7	34.7			36.9	44.7	36.5-37.5			6.9*
	Gibbs (1976)			24.0	43.3	26.0	13.4	35.2			4.7
3156 [b]	Panou (2016a)			23.86	38.9	33.1	44.5		14.1	10.9	
	Rayet (1875)				42.2						
	Gibbs (1976)			24.0	41.5	31.0	34.0	39.2			9.3
3157 [b]	Panou (2016a)			23.86	21.4	35.7	35.1		14.8	15.0	
	Gibbs (1976)			24.0	21.0	34.1	30.8	39.4			13.1
ST 148 [c]	Panou (2016a)			23.86	33.6	32.6	15.3		9.5	19.3	
	Gibbs (1976)			24.0	33.5	44.5		40.8			6.6
ST 209 [d]	Panou (2016a)			23.86	22.8	30.8			13.3		
	Gibbs (1976)			24.0	26.0	43.4	46.6	36.2			7.2
ST 147 [b]	Panou (2016a)			23.86	34.4	39.8	29.6		5.7	7.3	
	Gibbs (1976)			24.0	36.0	13.6	28.0	34.6			4.4
ST 780 [b]	Panou (2016a)			23.86	29.5	36.0	34.9		13.6	14.0	
	Gibbs (1976)			24.0	30.0	27.5	40.8	36.4			10.1

ε_1: angle (measured experimentally) formed by two rays emanating from the peak of the gnomon and ending to the dial plate at the intersection points formed by the winter solstice curve and the curve of equinoxes curves with the meridian line respectively; ε_1: angle (measured experimentally) formed by two rays emanating from the peak of the gnomon and ending to the dial plate at the intersection points formed by the summer solstice curve and the curve of equinoxes curves with the meridian line respectively; φ_1: calculated by measurements taken between the winter solstice curve and the curve of equinoxes; φ_2 calculated by measurements taken between the summer solstice curve and the curve of equinoxes; φ: measured experimentally; d_1, d_2, d: calculated using φ_1, φ_2, φ respectively (*present-day gnomon length); [a], [b], [c], [d]: sundial's category according to methodology followed for their study (see paragraph ''2. *METHODOLOGY*'')

4. DISCUSSION

4.1. Discussion on new measurements

The newly measured sundials (see Table 2) operate in the wider region of Athens (geographical latitude of Athens: 37.98°) except the sundial EM 2094. The sundial A 1870 can also be used as a time device in ancient Greek religious sanctuaries such as Dion (φ=40.2°) and Delphi (φ=38.5°).

For the sundial EM 2094 we conclude that it is not accurately constructed due to the fact that the calculated values φ_1 and φ_2 for the geographical latitude do not coincide. The value φ_1=43.7° corresponds to regions such as central Italy and central Balkans, while the value φ_2=57.9° corresponds to regions further north than central Europe. However, it can be used for measuring time with accuracy during winter months in central Italy or central Balkans. Due to its huge size, its decorative elements and its inscription 'NIKA' (means 'WIN') it is very likely that the sundial was a booty of war.

Lastly, the calculated values for the gnomon lengths are reasonable considering the size of their constructions.

4.2. Discussion on reassessed measurements

Firstly, we discuss the reassessed measurements on the sundials with gnomon and secondly the reassessed measurements on the sundials/ fragments with no gnomon. Then we compare and contrast our measurements with the others researchers' measurements (see Table 3).

4.2.1. Sundials with gnomon

Concerning the sundial 3158 we calculate for the first time the value of the obliquity of the ecliptic experimentally (ε_1=28.0° and ε_2=31.0°). The fact that these values are close to each other greatly enhances the use of the sundial as a precision device. The sundial operates at geographical latitude 42.0° throughout the year and the present-day gnomon length (5.9 cm) is appropriate considering the size of the marble construction. This means that the sundial operates in regions which latitudes are greater than those of northern Greece or for central Italy (Rome is located at latitude 41.9°) and is accurately for daily ordinary needs. The sundial has also been studied by Gibbs. She uses for her calculations the theoretical value of the obliquity of the ecliptic (ε=24°) and states that during the winter months the sundial operates at latitude φ_1=42.0° and during the summer months at latitude φ_2=34.5° respectively. Finally, she concludes that the sundial operates at 32.2° and the length of the gnomon is 5.97 cm. We made again Gibbs' calculations and the calculated value for the geographical latitude does not coincide with the value given by Gibbs. In Gibbs' calculations the latitude value should be 50.1° and not 32.2° as it is referred, a rather typographical error.

Concerning the sundial MP 1131 we first measure the inclination of the gnomon 18.87° and the length of its gnomon (6.8 cm). In order to apply the methodology for calculating the characteristic parameters (ρ, φ, d) of this sundial we assume that the gnomon has no inclination. Then, we calculate the values of the obliquity of the ecliptic experimentally (ε_1=20.0° and ε_2=32.7°). The fact that these values are not close to each other does not enhance the use of the sundial as a precision device. However, we continue our calculations and we conclude that during the winter months the sundial operates at latitude φ_1=57.5° and during the summer months at φ_2=41.7° respectively. These values correspond to regions further north of northern Greece. The appropriate gnomon length for the two latitudes is calculated d_1=4.1cm and d_2=3.7cm respectively, values slightly more than half of the value of the current gnomon length. Assuming that the sundial is used at Piraeus the ideal length of the gnomon should have been 5.1 cm (calculated using ε=23.86°). Consequently, the sundial is appropriate only for measuring time intervals in northern Greece areas during summer months and the original gnomon length of the sundial is about 4.0 cm. Thus, we conclude that the present-day gnomon of the sundial is added subsequently. The sundial has also been studied by Páris and Gibbs. Páris calculates the value of the obliquity of the ecliptic by measurements on the dial plate. He results that the construction of the sundial is not accurate (ε_1=24.72° and ε_2=34.73° values do not coincide) due to possible dialler's error. However, he mentions that the sundial is accurate for daily ordinary needs. Páris also measured the geographical latitude of operation of the sundial 36.5° ≤ φ ≤ 37.5° (a value close to the geographical latitude of Piraeus, Greece: 37.95°) from the outer geometric features of the construction. Sixty years later, Gibbs studies the sundial and uses for her calculations the theoretical value of the obliquity of the ecliptic (ε=24°). She calculates that during winter months the sundial operates at φ_1=26° and during summer months at φ_2=13.40° respectively. Finally, she concludes that the sundial operates at 35.20° and the length of the gnomon is 4.68 cm. However, both Páris and Gibbs do not mention how they deal with the slanting gnomon. If the gnomon of the sundial slants, Gibbs' methodology for south-facing conical sundials is not applicable. Páris calculates the geographical latitude both from measurements on the external lateral surface of the construction (φ=36.6°)

and from measurements on the dial surface (ε_1=24.7°, φ_1=36.9°; ε_2=34.7°, φ_2=44.7°). He concludes that the sundial is accurately constructed concerning the hour lines, the winter solstice curve and the curve of equinoxes as well. During summer months the daily hour can only be determined due to the fact that the φ_2 value corresponds to far northern latitude. Gibbs uses the theoretical value of the obliquity of the ecliptic for her calculations (φ_1=26.0°, φ_2=13.4°, φ=35.2°).

For sundial 3158, the contributions of our work in comparison to the previous research is that we first calculate the value of the obliquity of the ecliptic and the geographical latitude of operation of the sundial on the basis of in-situ measurements only.

For sundial MP 1131 the contributions of our work in comparison to the previous research are: a) the inclination of the present-day gnomon is 18.87° b) the sundial operates accurately at regions further north of northern Greece during summer months and c) the present-day gnomon of the sundial is added subsequently. The original gnomon length is about 4.0 cm.

4.2.2. *Sundials with no gnomon/fragments*

The main drawback of the sundials/fragments with no gnomon is that the value of the obliquity of the ecliptic cannot be calculated. Thus, we use the value ε=23.86° for calculating the characteristic parameters (ρ, φ, d) of the six reassessed sundials, while Gibbs use the value ε=24° for her calculations.

According to our measurements the curves and the hour lines have been well placed according to the latitude of design on the dial plate of the sundials No 3157 and ST 780. These sundials are accurate time devices ($\varphi_1 \approx \varphi_2$) because throughout the year they operate at the same latitude (35.0-36.0°). The regions they operate correspond to areas of the Aegean Sea, such as Rhodes, Karpathos, Kasos and Crete islands. The appropriate gnomon length of the sundials is calculated 14.9 cm and 13.8 cm respectively (see Table 3). Gibbs calculates that during winter months the sundial 3157 operates at φ_1=34.1° (southern Grete, Greece) and during summer months at φ_2=30.8° (Alexandria, Egypt) respectively. However, she states that throughout the year the sundial operates at φ=39.4° (central Greece, southern Italy) and the initial gnomon length is 13.1 cm. Gibbs' work on sundial ST 780 concludes that during winter months the sundial operates at φ_1=27.5° (central Egypt) and during summer months at φ_2=40.8° (north Greece, north Italy) respectively. However, she states that throughout the year the sundial operates at φ=36.4° (northern Peloponnese, north Cyclades, Dodecanese) and the initial gnomon length is 10.1 cm.

Our work on sundials 3156, ST 147 and ST 148 shows that the sundials operate at different latitudes throughout the year. The calculated values φ_1 and φ_2 are not close to each other so we cannot draw conclusions whether the sundials operate accurately within a particular period of time throughout the year or not. The calculated values for their gnomon length are not close to each other. As a result, these sundials are not accurate time devices and neither gnomon length can be determined. Gibbs' work on sundials 3156 and ST 147 concludes that in winter months sundials operate at different latitudes (φ_1) than in summer months (φ_1). Moreover, she concludes that the sundials operate throughout the year at latitudes (φ) greater than those in winter or summer months. In addition, Rayet calculated the angle between the axis and the generatrix ρ=42.2° and concluded that the conic section at the top edge is a hyperbola without making further calculations for the latitude and the gnomon length. Concerning the sundial ST 148 Gibbs states that throughout the year the sundial operates at different latitude than during winter months. She calculates the gnomon lengths based on the latitude value the sundials operate throughout the year.

Due to the fact that the dial plate between the summer solstice curve and the curve of equinoxes (along the meridian hour line) is broken, the geographical latitude of operation for the conical fragment ST 209 is calculated by measurements taken between the winter solstice curve and the curve of equinoxes. We calculated that during winter months the sundial operates at latitude 30.8° (northern Egypt). However, this value is not representative, hence, to draw conclusions about the manufacturing accuracy of the sundial.

Comparing our results for the characteristic parameters (ρ, φ, d) of the six sundials with the previous researchers' results we conclude that our results are more accurate than Gibbs'. Although both Gibbs and we apply the same methodology for south-facing conical sundials, the results are different. Firstly, Gibbs uses for her calculations ε=24°; we use ε=23.86°. Secondly, our measurements are more accurate than Gibbs' as we have already discussed.

5. CONCLUSION

The archaeoastronomical research here focus on sundials of conical type of Museums of Athens. Elaborately measurements are taken for first time and reassessed ones are compared to earlier investigations by others.

The majority of the sundials have been constructed for regions surrounding the Mediterranean Sea, where the ancient Greek civilization and culture flourished, with seven (7) out of fourteen sundials operated in locations throughout today Greece (A 1870, A 1769, ST 589, EM 2922, EM 9818, 3157, ST 780). These seven sundials are accurately constructed concerning the hour lines and the curves (winter solstice curve, summer solstice curve, curve of equinoxes) and served daily ordinary needs and astronomical time measurements throughout the year.

The rest seven (7) sundials/fragments (EM 2094, 3158, MP 1131, 3156, ST 148, ST 209, ST 147) are not accurately constructed. This is rather due to their incomplete or broken grid. However, based on our calculations we concluded that these sundials have been constructed for a broad range of regions such as central Balkans, central Italy and Egypt, confirming that the sundial is a widespread instrument of Greek antiquity.

Seven out of fourteen sundials (sundials A 1870, A 1769, ST 589, EM 2922, EM 9818, 3157, ST 780) could have been used to regulate the duration of religious ceremonies at Athens (φ=37.98^o), Dion (φ=40.2^o) and Delphi (φ=38.5^o) while five out of fourteen for measuring the duration of land/sea travel routes (sundials 3158, MP 1131, ST 148, ST 209, ST 147). Five out of fourteen sundials that are accurately constructed (sundials A 1870, A 1769, ST 589, 3157, ST 780) could have been used in antiquity for determining the summer solstice, winter solstice, equinoxes and the succession of seasons as well.

There is no doubt that sundials are cultural objects which are constructed grace to the knowledge of mathematics, geometry, sculpture, architecture, and astronomy. The concept of time is undoubtedly part of the daily habits and culture of the inhabitants of ancient Greece. The prediction of the succession of the seasons is vital for daily activities, festivities and especially the agricultural ones.

It should be emphasized that the accurate construction of these sundials denotes that the value of obliquity of the ecliptic is widely known in antiquity. Moreover, the sundials are constructions of historical and cultural importance due to their carved decorative ornaments and the engraved inscriptions.

The sundials should be considered today as cultural heritage objects for huge astronomical, social, historical and cultural value.

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