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ASTRONOMICAL ALGORITHMS OF EGYPTIAN PYRAMIDS SLOPES AND THEIR MODULES DIVIDER

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ABSTRACT

This paper is an attempt to show the astronomical design principles that are encoded in the geometrical forms of the largest five pyramids of the fourth Egyptian dynasty, in Giza and Dahshur plateaus, based on using the pyramids' design-modules that are mentioned in the so-called Rhind Mathematical Papyrus. It shows the astronomical algorithms for quantifying the slopes of pyramids, with reference to specific range of earth's axial tilt, within spherical co-ordinates system. Besides, it decodes the design of the ancient Egyptian cubit rods with reference to the systems of measurement used by Herodotus and Diodorus, and proves that Herodotus never talked about Giza pyramids and he meant only the pyramids of Memphis in Dahshur. In addition, it proves that the Egyptian arrow of 22.5cm, or half the mean cubit of 45cm, is the largest common divider of the design-modules of these pyramids, and accordingly it retrieves their intended astronomical design dimensions.

KEYWORDS: ***Giza pyramids, Dahshur pyramids, obliquity, axial tilt, Egyptian cubit, bent pyramid, red pyramid.***

1. INTRODUCTION

Methods of design and construction of the Egyptian pyramids were, and still are, in the forefront of scientific debates. In Egyptian mathematics, the so-called Rhind Mathematical Papyrus-RMP that was first translated by Thomas Eric Peet (1923), and then by Arnold Buffum Chace *et al* (1927-1929), is the oldest mathematical source book, where four of its eighty-four problems (RMP#56-59) show the design-sketches and slope-ratios of two pyramidal design models, i.e., the pyramid's height to half its side-length. In the architectonic and archaeological realms, there are five large pyramids found in Egypt from the days of the fourth dynasty, which still retain their original form, and are grouped in two sites: the three Giza pyramids, and the red and bent pyramids in Dahshur. According to Petrie (1883, 1887), the slopes of these pyramids are between 43^o and 55^o. Retrieving the design method that was used to set the pyramids' slopes with reference to spherical co-ordinates became the interest of modern scholars[1]. In archaeoastronomy, Belmonte and Magli (2015) proposed an idea similar to the shed design method, where they suggest that the slopes of these pyramids match the elevation angles of the ascending sun when it crosses the first vertical east-west plane and thus illuminate their four faces for few minutes, in specific days. Contrary to this opinion, previous publications by the author showed that the prime aim of the pyramids designers was to encode and preserve their scientific knowledge for the coming generations. For example, Aboulfotouh (2007) discussed the concept of shrinking contour circles that models the relative change of earth's axial tilt (obliquity), from an assumed lower limit (~21.672^o), as a frame of reference, in order to retrieve the ancient Egyptian relativistic algorithms that most likely were used to reckon the tilts of the entrance passages of these five pyramids. Related to this, Aboulfotouh (2005) showed that the cotangent of edge angle of the great pyramid in Giza equals the result of subdividing the obliquity angle of time ~24.10^o by the lower limit. However, the astronomical parameter of earth's obliquity-range, proposed by the ancient Egyptian designers, was never discussed in relation to the design modules of the large pyramids of the fourth dynasty, or in relation to the design modules of the four pyramids problems in RMP. Besides, in history, Herodotus (440BCE, pp.426-437) mentioned the dimensions of the four pyramids of Memphis, where one of them he called the great pyramid. Nowadays scholars of Egyptology believe that Herodotus mentioned the dimensions of the largest four pyramids in Giza Plateau; despite he did not talk about the Sphinx. In geography, Strabo (1932-1967, p.89) said that those pyramids are about 40 stadia (~ 6.3km) south of Memphis[2], which matches the location of the four pyramids in Dahshur. On the other hand, Diodorus (60BC-30AD, p.32, 63-3) mentioned the correct location of the largest three pyramids, in the Giza plateau, as 120 stadia from Memphis and 45 stadia from the Nile, but the dimensions he mentioned are less than those of the Giza pyramids, and are similar to those of the pyramids of Memphis. Perhaps he cited the dimensions given by Herodotus[3]. Today, scholars find difficulties in verifying the dimensions given by those historians with the survey data of Petrie (1883, p111), and many anti-Herodotus criticisms were published accordingly. For example, How *et al* (1913-1927) pointed out that Herodotus's figures of Giza pyramids' dimensions are inaccurate; despite, many parts of Herodotus's text indicate that he studied civil engineering. Besides, in stating the heights of the Egyptian pyramids, ancient and mediaeval historians alike were mentioning instead either the height along the inclined edge of the pyramid, or the height along the line from the crest of the pyramid to the midpoint of its side[4], which did mislead some scholars. The accurate lengths of measurement units used by Herodotus and Diodorus and their correlation to the Egyptian cubits are still unknown. Hence, the paper discusses four key points. Firstly, it shows the most likely astronomical design algorithms for quantifying the slopes of pyramids, and in relation to the pyramids-modules in RMP. Secondly, it shows that Herodotus never talked about, and defiantly never saw the Giza pyramids; and he meant only the pyramids of Memphis, in Dahshur plateau. Thirdly, it decodes the design of the ancient Egyptian cubits, and identifies the metric value of measurement units used by Herodotus and Diodorus. Fourthly, it identifies the largest common divider of the design-modules of pyramids of the fourth Egyptian dynasty, and retrieves their intended astronomical design dimensions.

2. PYRAMIDS DESIGN MODULES

Fig.1 shows the hieratic text of two of the four pyramids problems in RMP, namely: RMP#56-59. Based on reviewing its hieratic text (see: Chace *et al*, 1927-1929, plates 78-81) and the translation by Clagett (1999, pp.166-168), the data of pyramids' height h and their side-length l in cubits are: in RMP#56, h=250 and l=360, in RMP#57-58, h= (93+1/3) and l=140. However, in RMP#59, the hieratic text states that h=12, and l= 8, and not the contrary. Accordingly, the first observation is that these problems show two design models, and not only one as some scholars believed. Besides, in the hieratic text, the pyramids' slopes are not represented as a number of palms[5], but as abstract modules, written under the modular arc notation (see: Aboulfotouh, 2012, p.135). In RMP#56-58, the number of modules of pyramid's height h_m is constant and equal 7, and the number of modules in each side of the pyramid's square base l_m is variable and is less than 7 modules in these problems. Hence, as known, $l_m/2$ indicates the slope β of the pyramid's four faces; i.e., $\tan \beta = 2h_m/l_m$. In the hieratic text, $l_m/2$ is denoted by the Egyptian Arabic word *Saqet*[6] that we use in descriptive geometry and means the horizontal projection (run) of the inclined line. In RMP#56, $l_m/2 = (5+1/25)$, which imply β= 54° 14' 46.01''; and in both RMP#57 and RMP#58, $l_m/2$= (5+ ¼), which imply β= 53° 7' 48.37''. Contrary to opinions of scholars, e.g., Gillings (1982, p.186) and Clagett (1999, p.168), RMP#59 shows different model, where the number of modules of the pyramid's side l_m is constant and equal 7; and h_m is variable, and in this case it equals (10+1/2). Hence, $\tan \beta$= 10.5/3.5, which imply β= 71° 33' 54.18''. It is similar to the pyramidal part at the top of an Egyptian obelisk, or like the pyramid of Khonsu in Deir el-Medina, with its steep slope.

Figure 1: The hieratic text of RMP#56 upper, and RMP#57 lower; the pyramid diagram with horizontal rectangular base is explained in fig.2; and the double inclined lines in the upper diagram imply a pyramid with concave faces, where the outer line is its main slope; it is used with the permission of the British Museum.

3. PYRAMIDS DESIGN AND ASTRONOMICAL PARAMETERS

Aboulfotouh (2007) retrieved the relativistic equations for reckoning the tilt a of the entrance passages of the largest five pyramids of the fourth Egyptian dynasty. The equation of the great pyramid in Giza plateau is: $\sin a = (\sin \lambda * \sin O_m)/(1-(O_i^2/O_t^2))^{1/2}$; where its a = ~ 26.56°. Here, λ is the latitude ~30° at which the pyramid stands. Additionally, Aboulfotouh (2002, 2005, 2014) showed that the encoded axial tilt[7] O_t (or ε) in the design of the great pyramid and the site plan of the horizon of the Giza pyramids is ~24.10°; where its designer used an obliquity range from O_i= ~21.672° as lower limit to O_x=~24.30° as upper limit, and O_m= ~22.986° as a mean. Besides, using the arcs' lengths of the obliquity-angles as radii of contour circles, Aboulfotouh (2007) showed that $O_i/O_t = \cos \varphi$, where φ is the angle of the horizontal deviation of time that equals ~25.93° for O_t=24.10°, which is used as tilt of the upper entrance passage of the 2nd pyramid in Giza plateau. The tilt of its lower entrance passage was found equal to O_i= ~21.672°.

Furthermore, using the data of Petrie (1883, p.43), Aboulfotouh (2005) showed that $O_i/O_t = h/r$, where h of the great pyramid is ~146.51m; and r is radius of the circle that encloses the pyramid's square-base, i.e., $r = (l/2^{1/2})$. For l = 230.35m., r = 162.88m. Besides, it was shown that r is also the radius of the spherical co-ordinates system of the pyramid (see fig.2) upon which its internal-design was created. That imply, $\cos \varphi = h/r = \tan \psi$, where ψ is the pyramid's edge angle. Using the equation: $h = r\,(O_i/O_t)$, if we intend to construct another great pyramid to encode O_t=23.452145°, i.e., ε of the epoch 1900AD (Wittmann, 1979), for r =162.88m, its h would be ~ 150.51m.

Fig.2 shows north-south cross section in a spherical co-ordinates system, where point C represents the earth, line Ys-Yn represents the earth's axis of rotation, line P_1-P_2 represents the plane of the ecliptic, and line Xe-Xw represents the plane of the equator. Ahoulfotouh (2005, 2014) showed that this cross section was used for creating the internal design of the great pyramid as well as the site plan of the horizon of Giza pyramids.

In fig.2, the generated main-rectangle P_1-D_1-P_2-D_2 includes the triangle cross-section of the pyramid M_1-Z-M_2 and the residual-rectangle P_1-G_1-G_2-D_2, similar to the hieroglyphic sign that denotes a pyramid △. The horizon line H-H divides the main-rectangle into two equal parts; the upper half is V_1-D_1-P_2-V_2. If we assign names for the vertical distances P_2-V_2, V_2-G_2 and G_2-D_2 as: j, k, and f, respectively; and taking into consideration that $J= f+k$, the pyramid's height $h= k+J$. Moreover, the altitude k of center of the pyramid's spherical system C (and the horizon line H-H) from its zero ground-level in the construction site; and the tilt of its entrance passage a will vary according to the latitude λ where the pyramid stands, and also according to the used O_t. The altitude k could be reckoned using the equation: $k= h- (sin\ (90- \lambda- O_t)* r)$. Therefore, this pyramid design-model is latitude and axial-tilt identifier, i.e., a space-time pyramid. In the great pyramid, f= ~44.5m., k= ~51m., and J= ~95.5m, where its k is close to the level of the highest point of its grand gallery (Aboulfotouh, 2005). If we imagine that this pyramid will not collapse and keep standing during one complete obliquity cycle, over time, it is expected that k will fluctuate between two limits. The upper limit is k_x for O_x and the lower limit is k_i for O_i; e.g., its k_i=45.45m for $O_t = O_i$ =21.672º, and its k_x =51.42m for $O_t = O_x$ =24.30º.

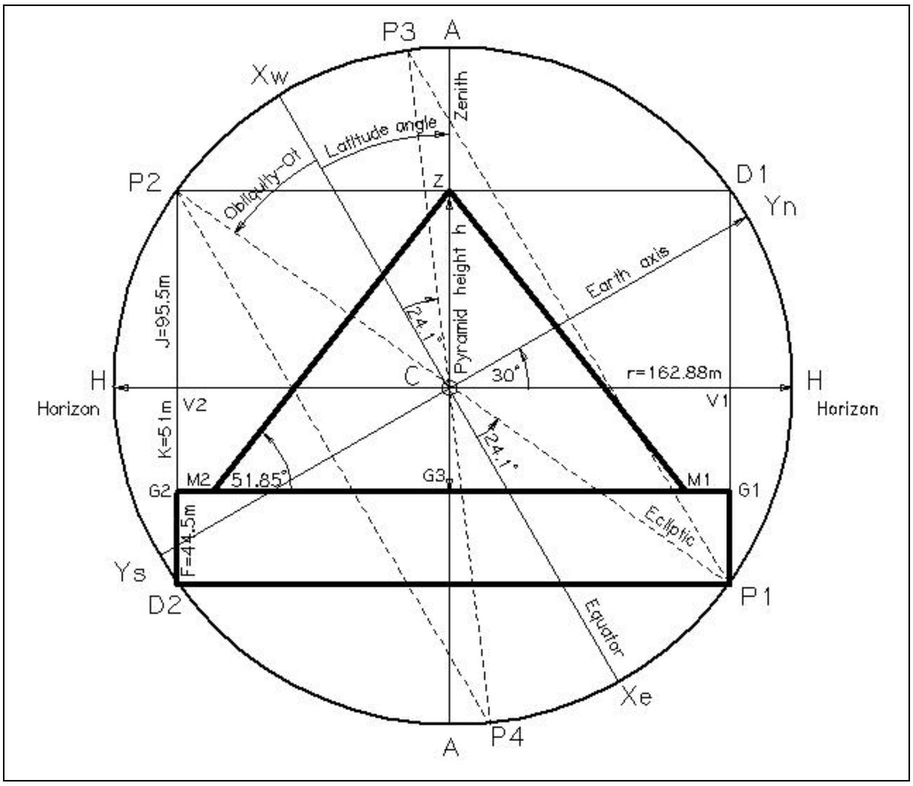

Figure 2: North-south cross section in a spherical co-ordinates system that shows the astronomical design parameters of the Great Pyramid in Giza Plateau, while looking due west.

Based on reviewing the diverse slopes of the Egyptian pyramids, the author identified four design models. Their astronomical slope-ranges are symbolized here below as: β_1, β_2, β_3, and β_4. These ranges of pyramids slopes are reckoned, using the assumption of the ancient Egyptian designers concerning the descending values of earth's axial tilt O_t (or ε), from ~24.30º as upper limit to ~21.672º as lower limit.

In the first model, since $h/r=O_i/O_t$; $l/2= r/(2^{1/2})$; and $tan\ \beta_1= 2h/l$; then, $\beta_1= Atan\ (O_i*(2^{1/2}))/O_t$. Accordingly, the lower limit of β_1 is ~51º 35' 26.81", and its upper limit is 54º 44' 8.2". Using the data of Petrie (1883 & 1887), the slopes of the three Giza pyramids and the lower part of the bent pyramid are within the range of β_1. The slopes of pyramids models in RMP#56-58 are also within this range.

In the second model, the pyramid's slope β_2 could be set equal to the edge angle ψ of the first pyramidal model. That is, by using the diagonal cross-section of the first model as the north-south cross-section of the second model, taking into consideration that, the real radius r of the circle-enclosing the pyramid's square base, and its spherical co-ordinates system, is variable. In this case, $\beta_2= Atan\ (2h/l)= Atan\ (O_i/O_t)$. Accordingly, the lower limit of β_2 is ~41º 43' 41.63" and its upper limit is 45º. Using the data of Petrie (1887, pp.27-32), the slope of the red pyramid and the upper part of the bent pyramid are within the range of β_2.

In the third model, the pyramid's slope β_3 could be set equal to the angle that complements the deviation angle of time φ, i.e., $\beta_3 = Asin\ (90º- \varphi)= Asin$

(O_i/O_t). Accordingly, the lower limit of β_3 is 63° 6' 24.77" and its upper limit is 90°. For example, β_3 = 70° 33' 41.16" for O_t= 22.986°. The slope of the pyramid-model in RMP#59 is within the range of β_3.

In the fourth model, the pyramid's slope could be set as β_4= *Asin* (O_i/φ), where φ= *Acos* (O_i/O_t). This case implies that the deviation angle of time φ is used as radius of shrinking circle, and is being referenced to the circle that its radius is the lower limit of earth's obliquity O_i (see Aboulfotouh 2007). For O_i =~21.672°, the possible range of O_t is from ~24.30° to ~ 23.3205°. Accordingly, the lower limit of β_4 is 53° 41' 34.0" and its upper limit is 90°. The recorded slope of pyramid of Unas[8] is within the range of β_4; where it is close to 56° 39' 52.77" that corresponds to O_t=24.10°.

Petrie measured the side length l of the three pyramids in Giza plateau and reckoned the height of each using what he recorded as mean slope for each pyramid. He recorded 230.35m and 215.26m as l of the great and 2nd pyramids in Giza plateau, respectively; and recorded 51° 52' ±2' and 53° 10' ±4' as their β_1, respectively (Petrie, 1883, p.43 & p.98). His data confirm the values of O_t for these two pyramids, as 24.10° and 22.986°, respectively. Regarding the 3rd pyramid, he recorded 105.5m as l of its base. However, either due to the demolition of the granite casing of the 3rd pyramid[9] during the 13th century AD, or due to its concave faces, as in RMP#56 in fig.1, Petrie (1883, p.111) recorded four different slopes: 50° 57', 50° 42', 51° 0', and 51° 58'. Accordingly, he assumed that 51° 0' ±10' is its most probable slope. None of his four records encodes the obliquity angles that are used by designers of the fourth dynasty. Most probably, the intended β_1 of the 3rd pyramid is 51° 33' 12.41" that corresponds to O_t=24.30°, as what is encoded in the tilt of its entrance passage (Aboulfotouh, 2007).

Furthermore, according to Petrie (1887, p.27&p.32) the slope of the core masonry of red pyramid is 44° 36', and he said Vyse stated this as 43° 36' 11". Perhaps, Petrie did not notice the concaveness in its four faces[10]. Despite 44° 36' is within the astronomical range of β_2, it does not imply encoding any of the obliquity values that are used by designers of the fourth dynasty. In addition, 45° is not valid because it will increase its h for l=220m. Most properly, its intended β_2 is 43° 18' 52.7" that corresponds to O_t=~22.986°, as what is encoded in the tilt of its entrance passage (Aboulfotouh, 2007). Besides, Petrie (1887, pp.28-30) measured the slope of the lower part of the bent pyramid and recorded diverse values from 54° 36' to 55° 1'. Most probably, the intended design slope of the lower part is the upper limit of β_1, i.e., 54° 44' 8.2". He recorded 189.45m as l of its base, and concerning the horizontal-projection of the slope of its lower part, he recorded two distances: 32.7m and 33.0m. Concerning the slope of the upper part of the bent pyramid, he recorded 43° 0' and in the west side, he said, "it is perhaps 42° 39' higher up". Most probably the intended slope of the upper part is 42° 54' that encodes O_t=~23.3205°, and identifies the lower limit of the deviation of angle of time φ, i.e., at this limit φ=O_i=~21.672°.

By reviewing the design of the bent pyramid, it seems that its designer combined the first and second models in one pyramidal form. The final product shows only the lower part of the model β_1, and the upper part of the model β_2. Despite the rest of the two models are not observed, their r, l, and h could be identified, using the survey data of Petrie (1887, pp.28-30). In this special case, apparently, its designer used other altitude k_φ related to the deviation angle of time φ, where $k_\varphi = r - (\sin(90 - \lambda - \varphi) * r)$. In fig.2, the distance, AZ will equal k_φ, when O_t is put equal φ; and k_φ will be measured from the ground point G_3. The altitude of the bent line[11] of the bent pyramid would be ~46.5m, for the lower horizontal-projection, between 32.7m and 33.0m, and the intended β_1= 54° 44' 8.2". The altitude of the bent line matches k_φ=46.51m of its model β_1, for r=h =133.875m, λ = 29.788°, and φ = 0.0°. Besides, in its model β_2, most probably its intended radius r=158.19m, and its intended h= 103.95m, where it is the observed height of this combined pyramid; and the observed l of its base= 133.875*$2^{1/2}$= 189.33m. Table.3 shows the intended design data for this combined pyramid.

4. PYRAMIDS DESIGN AND MEASUREMENT UNITS

In the realm of architecture, designers state the dimensions of buildings in three ways: (i) in terms of design modules, (ii) in terms of specific measurement-unit to be used for implementation, and (iii) in terms of other measurement-unit that might was used for survey, after implementation. In the realm of megalithic construction, modern surveys, using any measurement unit, belong to the third category; and it might not imply knowing the original design module or the measurement unit used for design and implementation. In all cases, particularly after thousands of years, the results will include some deviations from the original design dimensions, in drawings. The first is at least the allowable implementation tolerance, if not exceeded it. The second is due to other vertical or horizontal force of seismic waves that might have caused, e.g., displacement, cambering, deflection, twisting, and/or tilting of all or part of the megalithic structure. The third is partial demolition due to human vandalism or by natural effects that create the second cause. In pyramids design realm, it is necessary to understand the design principles of the ancient Egyptian cubit rods. Sarton (1936, pp.399-402) noted out that the cubit rod of 28 digits is 52.5cm, and referring to plate-I by Lepsius (1866, p.18), from right to left, he said the first 16 digits are 18.75mm long, the eight following are 17.19mm long, and the last four are 21.87mm long. Correlating these numbers, the last two figures should be 17.1875mm and 21.875mm, to become 7/6 and 11/12 of the mean-digit δ (18.75mm) respectively. Hence, its design length[12] is exactly 52.5cm. Besides, starting from the right side, the first fifteen mean-digits are subdivided to create mini-intervals, starting from 1/2, 1/3, 1/4,…, down to 1/16 of δ=18.75mm, respectively. By reviewing the design of four types of ancient Egyptian cubit-rods of the same size, it seems that all cubit-rods contain three types of digits; one of them is the mean-digit δ (18.75mm). The diversity remains in changing the sizes of the other two types of digits: above and below the mean δ, as well as their number and location in the cubit-rods[13]. Fig.3 shows the design components of the ancient Egyptian measurement rod of 52.5cm, the so-called the royal cubit of Torino.

Besides, in some cubit rods, we see special hieroglyphic marks that identify the limit, or the domain, of lengths, e.g.: the bird's claw implies a foot; claw with sparrow imply a short foot; an arrow looks like reed pen imply half a cubit; and hand with sparrow imply a short cubit. In general, a palm is four digits; an arrow is 3 palms, a foot is 4 palms; 1.5 foot or 2 arrows make a cubit. Thus, the number of digits in any cubit is only 24. The 28 mean digits and their 15 types of mini-intervals are the design grids for creating diverse sets of 24 fingers that form the lengths of diverse types of cubits, including their related feet, arrows, and palms, in the same measurement rod. For Example, (i) cubits of 39.375cm, 41.25cm, 45cm, and 52.5cm, (ii) feet of 26.25cm, 27.5cm, 30cm, and 35cm, (iii) arrows of 19.6875cm, 20.625cm, 22.5cm, and 26.25cm, (iv) palms of 6.5625cm, 6.875cm, 7.5cm, and 8.75cm. In this context, the cubit of 45cm long is the mean cubit[14], which is composed of two arrows of 22.5cm. In some rods, the arrow symbol takes the digit number 12, to mark its mean length of 22.5cm; and in other types, it takes the digit number 13, to become 24.375cm long, or ½ a cubit of 48.75cm.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2.1875*4= 8.75cm				1.71875*8= 13.75cm								1.875*16 = 30 cm															
larg palm				small palm				short foot			arrow	arrow	foot	cubit							short cubit				royal cubit		
2.1875	2.1875	2.1875	2.1875	1.71875	1.71875	1.71875	1.71875	1.71875	1.71875	1.71875	1.71875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875
													16s	15s	14s	13s	12s	11s	10s	9s	8s	7s	6s	5s	4s	3s	2s
			8.75	6.875							22.5	24.375	26.25							39.375	41.25		45		48.75		52.5
			1/6 royal cubit				1/4 foot of 27.5cm				One arrow of 22.5cm	One foot or one arrow	One foot of 26.25cm							1.5 foot of 26.25cm (one cubit)	1.5 foot of 27.5 (one cubit)		2 arrows of 22.5cm (mean cubit)		2 arrows/feet of 24.375cm		One royal cubit of 52.5cm

Figure 3: The architectonic design of the Egyptian measurement rod that contains diverse types of cubits, foots, arrows, palms, and fingers; its cubits starts from the cubit of 39.375cm up to the royal cubit of 52.5cm, where each cubit contains only 6 palms. Its design direction is from left to right, and for geometrical use, it will be turned upside-down; in order to make the first subdivided digit of 2s in the left side of the architect/engineer, and below the intended drawing line. This integrated design of the Egyptian measurement rod of diverse cubits implies that it might was used in many parts of the known world in their days, similar to the rod of diverse-scale ratios that we use today.

Correlating the design principles for producing the Egyptian cubit rod that mentioned above with reliable historic reference, might reveal some ancient measuring systems. Herodotus (430BCE, p.559, 2-149) correlated the measurement units that he used as follows: (i) one hundred fathoms is exactly a stadium (stade) of six hundred feet; (ii) the fathom is six feet in length, or four cubits; (iii) a cubit measures six palms, and a foot is four palms; (iv) parasang is thirty stadia; and (v) schoene is sixty stadia. Using these correlations between measurement-units, table.1 shows the possible metric values of each, based on using four types of feet that are included in the design of the Egyptian measurement rod. Besides, Herodotus (430BCE, p.285, 2-9) said, "the distance between Heliopolis and the sea is fifteen hundred stadia, and sailing up the river from Heliopolis to Thebes, the distance is eighty-one schoenes". Using Google Earth Data, of 2015, these two distances, along the course of the Nile River are about 250km and 670km, respectively. This implies that Herodotus used the measurement system based on the foot of 0.275m, i.e., he used the stade of 165m, and the schoene of 8,250.0m.

Moreover, Diodorus (60BC-30AD, p.24, 47-1) said, based on citing Hecataeus, "ten stadia from the first tomb of concubines of Zeus (i.e. entrance of the valley of the Queens), stands a monument of the king known as Osymandyas (i.e. Ramesseum temple), and its first pylon is two plethra in breadth". Using Google Earth data (2015), this walking distance, along the road, is about ~1.89Km, and the breadth of the first pylon of the Ramesseum temple is ~63m. Besides, Diodorus (60BC-30AD, p.26, 51-5) said, "ten schoene above Memphis king Moeris (Seti-I) excavated a lake". Using Google Earth data, the distance from Memphis to the old entrance of Lake Moeris (beside Lahun Pyramid) is ~78km. It implies that Diodorus used the measurement system based on the foot of 0.2625m, i.e., he used the stade of 189m, the plethra of 31.5m, and the schoene of 7,875.0m. In this regard, Nallino (1910, pp.268-275) said[15], "Eratosthenes used the stade of Alexandria of 157.5m long", which equals 600 feet, each is 0.2625m long.

Table 1: The possible metric values of diverse measurement units mentioned by Herodotus.

Name	(mean) Length in meter	(large-royal) Length in meter	(small) Length in meter	(short) Length in meter	Design intervals
Finger (δάκτυλος)	0.01875 (mean δ)	0.021875 (7/6 of δ)	0.0171875 (11/12 of δ)	0.01640625 (7/8 of δ)	1/16 foot (1/24 cubit)
Palm (παλάμη)	0.075	0.0875	0.06875	0.065625	1/4 foot (1/6 cubit)
Arrow	0.225	0.2625	0.20625	0.196875	3 palms (1/2 cubit)
Foot (ποδες)	0.30	0.35	0.275	0.2625	4 palms (2/3 cubit)
Cubit (πήχηείς)	0.45	0.525	0.4125	0.39375	6 palms
Fathoma (οργιά)	1.8	2.1	1.65	1.575	6 feet
Plethra (πλέθρον)	30	35	27.5	26.25	100 feet
	36	42	33	31.5	120 feet
	39	45.5	35.75	34.125	130 feet
Stade (στάδιον)	150	175	137.5	131.25	500 feet
	180	210	165	157.5	600 feet
	216	252	198	189	720 feet
Parasang (παρασάγγες)	4,500.0	5,250.0	4,125.0	3,937.5	30 Stadia of 500 feet
Schoene (σχοίνον)	9,000.0	10,500.0	8,250.0	7,875.0	60 Stadia of 500 feet

Furthermore, Herodotus (430BCE, pp.426-437) mentioned the dimensions of the pyramids of Memphis[16], that Strabo (1932-1967, p.89) said, "they lie about forty stadia (6.30 Km.) south of Memphis". This location is the plateau that includes the pyramids of Dahshur, where its elevation is about a hundred feet high[17], or 27.5m. Despite Herodotus did not mention the Sphinx and the remaining small pyramids in the Giza plateau; scholars of Egyptology have thought that he describes the Giza Pyramids. Herodotus called the bent pyramid, as the pyramid of Cheops[18]. Concerning the dimensions of its existing causeway (of 690m in Google Earth data 2015), he said, "it is five stadia in length (137.5m*5), ten fathoms wide (16.5m), at the highest part, eight fathoms; and the pyramid is a square, eight plethra[19] each way, and the height (along its inclined edge) the same"[20]. The pyramid of Cheops's daughter (satellite pyramid), which lies directly south of, and close to, the bent pyramid, and each side of it measures one and half plethra[21]. The red pyramid, he called the pyramid of Chephren, and said, "it was built of the same bigness as that of Cheops, save that it falls forty feet (40*0.275=11m) short of it in height (along its inclined edge)"; and he added, "I have myself measured it". It implies that the length that Herodotus assigned to the side of the pyramid of Cheops (bent pyramid) is the result of measuring the base of the red pyramid, assuming that their square-bases are equal in size. The black pyramid (now called pyramid of Amenmhat-III) he called the pyramid of Mycerinus or Rhodopis[22] and said, "the length of each side of its square base falls short of three plethra by twenty feet"[23]. Diodorus (60BC-30AD, pp.32-33, 63-3 & 64-2,7) said, "the lengths on each side of the bases of pyramids of Chemmis (Cheops), Cephres (Chephren), and Cherinus (Mycerinus) are: seven plethra, a stade, and three plethra, respectively". Concerning the names of builders of pyramids of Memphis, Diodorus (60BC-30AD, 64-13) said, "with regard to the pyramids there is no complete agreement among either the inhabitants of the country or the historians; for according to some the kings mentioned above (in his text) were the builders, according to others they were different kings". Herodotus (440BCE, p.431, 2-128) also said, "people call these pyramids after the shepherd Philitis". Table.2 shows the dimensions of the pyramids of Mimphes (in Dahshur), by Herodotus and Diodorus, using different types of plethra, in comparison with the available data by Petrie and other scholars.

Table 2: Bases dimensions of Dahshur Pyramids by ancient and modern scholars.

Pyramids of Memphis, in Dahshur	Herodotus		Diodorus		Petrie[24] & Rossi[25]
	Plethra (πλέθρων)	meter	Plethra (πλέθρων)	meter	meter
Red Pyramid	**8** * (27.5m)	220	**7** * (31.5m)	220.5	220.5
Bent Pyramid	Assumed = to Red pyramid	--	**6** * (31.5m) or (stade)	189	189.56
Satellite Pyramid	**1.5** * (35.75m)	53.625	--	--	52.42
Black Pyramid	(**3***35.75m) – (20 *0.275m)	101.75	**3** * (34.125m)	102.375	105

5. THE LARGEST DIVIDER OF PYRAMIDS MODULES

Based on the above discussion, the common divider of pyramids of the fourth dynasty could be reckoned by dividing *h* of each pyramid by 7 and then finding out what is the size of the palm that divides the intended design modules of the five pyramids. Trying out diverse types of palms, the result was the mean-palm of 7.5 cm. This imply that each module in the five pyramids is composed of a number of mean-palms; if it is not a prime number, there might be a divider larger than the size of the mean-palm. For example, each module in the great pyramid is composed of 279 mean-palms, where 3 is the only integer from 2 to 7 that divides it. That is, the Egyptian arrow of 0.225m is the largest module divider *d* for designing the great pyramid, where it is 2/3 of the mean-foot of 0.30m and ½ of the mean-cubit of 0.45m. The case is also similar in the other four pyramids. Table.3 shows that the Egyptian arrow of 22.5cm is the largest common divider *d* of the design modules of pyramids of the fourth dynasty. It shows also, the design modules *m*, the number of module divisions n_d, the related intended design lengths (*h*, *l*, *k* and κ_φ), and the astronomical design parameters (O_i, O_t and φ) of each pyramid. Similarly, one can identify *d* in each of the four pyramids problems in RMP, using the royal cubit of 0.525m, in order to identify the metric lengths of pyramids in the-

Table 3: The intended design dimensions and astronomical parameters of pyramids of the fourth dynasty.

Pyramid	Modules of *h* and *l* (*m*)		Divisions in one module (n_d)	Intended design length ($m*n_d*d$) d=0.225m in meter	Astronomical parameters for O_i=21.672°; where O_t, and φ are in degrees; and κ and κ_φ are in meter			
					O_t	φ	κ	κ_φ
Great Pyramid	*h*	7	93	146.475	24.10°	25.93°	50.9	71.6
	l	11		230.175				
2nd Pyramid	*h*	7	91	143.325	22.986°	19.46°	51.8	53.2
	l	10+1/2		214.9875				
3rd Pyramid	*h*	7	42	66.15	24.30°	26.89°	22.9	33.8
	l	11+1/10		104.895				
Red Pyramid	*h*	7	66	103.95	22.986°	19.46°	9.66	54.2
	l	14+1/2+1/4+1/10		220.5225				
Combined or Bent Pyramid	$h_{\beta1}$	7	85	133.875	21.672°	0.0°	50.4	**46.5**
	$l_{\beta1}$	9+1/2+1/3+1/15		**189.3375**				
	$h_{\beta2}$	7	66	**103.95**	23.3205°	21.672°	36.8	42.1
	$l_{\beta2}$	15+1/20+1/63		223.7282143				

se problems that are mentioned in section-1 above, taking into consideration that in RMP#56-58, h_m=7, and in RMP#59, l_m=7. In RMP#56, d=0.15cm (1/2 mean-foot), n_d=125, and h= 0.15*125*7= 131.25m. In RMP#57-58, d=0.35m (royal foot), n_d=20, and h=0.35*20*7=49m. In RMP#59, d=0.30m (mean foot), n_d=2, and l= 0.30*2*7= 4.2m.

6. CONCLUSION

This paper showed four astronomical design algorithms for quantifying the slopes of pyramids, in relation to the pyramids design modules of RMP; and that could be used on earth or elsewhere, using diverse obliquity limits. It proved that Herodotus never talked about, and defiantly never saw, the Giza pyramids; and he meant only the pyramids of Memphis, i.e., Dahshur pyramids. It decoded the design of the ancient Egyptian cubit rod of 52.5cm, and showed that it includes other types of cubits, feet and arrows. It showed that Herodotus used the measuring system based on the foot of 27.5cm, and Diodorus used other system based on the foot of 26.25cm. It also, showed that the arrow of 22.5cm (half cubit of 45cm) is the largest common divider of the design modules of pyramids of the fourth Egyptian dynasty. In general, this paper showed an additional proof to support that defiantly there was consistent astro-mathematical theory in pyramids design (the Horizon theory), which was formulated and architectonically encoded by designers of pyramids of the fourth dynasty in ancient Egypt.

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NOTES

1 Without some basic knowledge in spherical astronomy, it is hard to retrieve any mathematical method for designing the slopes (angles of inclination) of the Egyptian pyramids. RMP includes mere numerical ratios without the design method. In her review, Spence (2002) disputed the strange conclusion by the mathematician Herz-Fischler who said in his book, "there may have been no true theoretical basis for the angles of inclination of pyramids".

2 Strabo said, "on proceeding forty stadia from the city (from Memphis heading south as he walks), one comes to a kind of mountain-brow; on it are numerous pyramids the tombs of kings, of which three are noteworthy; and two of these are even numbered among the Seven Wonders of the World, for they are a stadium in height."

3 Eyre, C. J. (1991, pp.223-225) said Herodotus's text mercilessly plagiarized by Diodorus Siculus.

4 How *et al* (1913-1927, p. 250, ii 124) called it «height along sloping side»; in the pyramids chapter, Al-Maqrizi (1849) said, based on citing Abdulrhim Alquise, "the perimeter of the Great Pyramid's base is 2000 black cubits and its height is 500 black cubits", implying the height along its inclined edge.

5 In the opinion of scholars, e.g., Gillings (1982, p.186) and Clagett (1999, pp.166-168), a cubit measures 7 palms, and thus they thought it represents the rise of the pyramids slopes in RMP. That is despite, Sarton (1936, pp.399-402) had noted out that the 7 palms shown in the cubit rods are not equal, and Herodotus (430BCE, p.559, 2-149) said, "a cubit measures only 6 palms".

6 Gillings (1972, p.185) pronounced it as *Seked*, and said, "it means the slope of the sides of a pyramid".

7 Using flat mutation of 0.47" per year from the epoch 2000 AD (23.45°), the obliquity 24.10° meets ~ 3055-3065 BCE; and using, M. Bessel's Eq.: ε = 23° 28' 18" + 0.48368" T – 0.00000272295" T^2 (Nallino, 1911, p.270) & (The Penny Cyclopedia, 1840, Vol. XVIII, p.495), 3065BCE meets ε= 24.109°; where ε is O_t, and T is the years elapsed from the epoch 1750AD. Bessel's equation is the only polynomial fit that gives value close to the record of Chou Li in 1100BC, ε=23.881° (data in: Wittmann 1978). The polynomial fits that are set to finite limits, e.g., between 22.5° and 24.5° (Meeus, 1991, p.135) or between 22.61° and 24.23° (Laskar, 1986, p.86), give ε=~24.027° for 3070BCE; and fail to give ε = ~23.922° for the azimuth 296.75° of winter solstice of the axis of the main Karnak temple, at λ=25.72°, in ~ 1620BCE (data in: Shaltout *et al*, 2005, p.15).

8 Its recorded slope is 56° 18' (Rossi, 2003, p.249; based on citing, Lauer, Observations, p.95).

9 Al-Maqrizi (1849), based on citing Abdelatif Al-Bugdady, said, "the demolition of the granite casing of the small pyramid was by the order of king Al-Aziz Osman in 593H, whose illiteracy made him imagine that there are treasures beneath it, but the lack of finance obliged the workers to discontinue after 8 months of demolition work".

10 Since, its h= ~104.61m that is composed of 154 steps (The Penny Cyclopedia, 1840, Vol. XIX, p.152), and its l=220.5m (Rossi, 2003, p253), any record above that of Vyse 43° 36' 11" is invalid.

11 Petrie (1887, p.32) reckoned it as follows: tan 55° 1' * 33m= 47.15m.

12 Over time, shrinking might cause slight reduction in the length of wooden rods; thus, any royal cubit length other than 52.5cm, e.g., 52.375cm or 52.3cm (Scott, 1942, p.70), is incorrect and misleading.

13 Further, 13/12 of the mean δ produces the cubit of 48.75cm; and 9/8 of δ produces the cubit of 50.625cm; 10/9 of δ produces the cubit of 50.0cm; and 61/60 of δ produces the cubit of 45.75cm that 2/3 of it is the English foot of today.

[14] Scott (1942, p.70) called it a short cubit, and Rossi (2003, p60) called its half (arrow) of three mean palms as small-hands. Scott (1958, pp.205-214) also mentioned the lengths of other cubits, e.g., 39.6cm and 50.0cm.

[15] Carlo Nallino (p.274) referred to the works of the German scholar Fr. Hultsch.

[16] In Greek: πυραμίδας κατατμηείσαι τας ἐν Μέμφι (Herodotus, 430BCE, p.282, 2-8).

[17] Herodotus (430 BCE, p.430-431, 2-127) perhaps mentioned the elevation from the plain of Memphis or from the water surface of the Nile River; and considering the metric value of the foot he used, Google Earth data of 2015 show the same elevation.

[18] Despite, Africanus had cited Manetho and said, "the fourth dynasty kings are: Soris, Suphis (i), Suphis (ii), Mencheres, etc." (Manetho, 1940-1964, p.47), Egyptologists thought that Cheops (or Chmmes) is the 2nd king of the 4th dynasty. Herodotus (430BCE) put Cheops the 4th king after Sesostris, and Diodorus (60BC-30AD, p.32) put him the 5th king after Sesostris, who most probably is the so-called Ramses-III, where Sesostris is a royal or military title before the king's cartouche. The names: Chemmis, Cephres, and Cherinus, match the cartouches of the so-called Ramses-VIII, Ramses-IX, and Ramses-XI, respectively.

[19] In Greek: ὀκτώ πλέθρα (Herodotus, 440 BC, p.426, 2-124).

[20] Strabo (36BC-24AD, p.89, 17-I, 33) said, "high up approximately midway between the sides, it has a movable stone, and when this is raised up there is a sloping passage to the vault"; related to this, see Fig-XI: Door of the South Pyramid of Dahshur, by Petrie (1883).

[21] In Greek: ὀλον καὶ ἡμίσεος πλέθρον (Herodotus, 440 BC, p428, 2-128).

[22] Most probably, Rhodopis is the so-called Nubheteti-Khered that her tomb is found under this pyramid; the correct pronunciation of the last word  in her name is Radophi (or Pharida) without the Greek "ς"; the upper letter is *Ph* not *Kh*, which becomes the last letter if we follow the ancient Egyptian spiral sequence of writing the letters of words.

[23] In Greek: εἰκοσι ποδῶν καταδέουσαν κῶλον ἑκαστον τριῶν πλέθρων (Herodotus, 440 BC, pp.436-437, 2-134).

[24] Data of the bent and satellite pyramids are by Petrie (1887, p.28).

[25] Data of the red and black pyramids are by Rossi (2003, p.253, based on citing, Stadelmann, MDAIK 39, p.253, and Arnold, Amenemhet III., p.9, respectively).