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THE MATHEMATICS OF PYRAMID CONSTRUCTION IN ANCIENT EGYPT

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ABSTRACT

This paper is concerned with the Mathematics associated with Pyramid construction in ancient Egypt from information preserved in Mathematical Papyri. An account of the evolution of the burial monument from the mastaba through the step pyramid to the true pyramid is given. The remarkable accuracy in the construction of right angles and the orientation of the pyramids is explored with reference to the Pyramid of Cheops (Khufu). The notion of the 'seked' (From the Mathematical Papyri of the Middle Kingdom) and its use in pyramid construction is explained. A theory as to how the volume of a truncated pyramid might have been deduced is presented. The study of the pyramid in Demotic Papyri is also mentioned.

KEYWORDS: pyramid, slope, orientation, mathematics, Egypt

THE EVOLUTION OF THE PYRAMID

Cheop's pyramid in Giza was considered as one of the seven wonders of the ancient world, and it is the only one that still exists. Yet, quite a few pyramids still survive not only in Giza, but also in Dahshour, Abussir, and other places (Fig. 1). The pyramid as burial monument of Pharaohs took its final shape after a long architectural evolution. The faith of ancient Egyptians in life after death led them to construct special types of tombs (Spencer 1982; Edwards 1993[9]; Borchardt

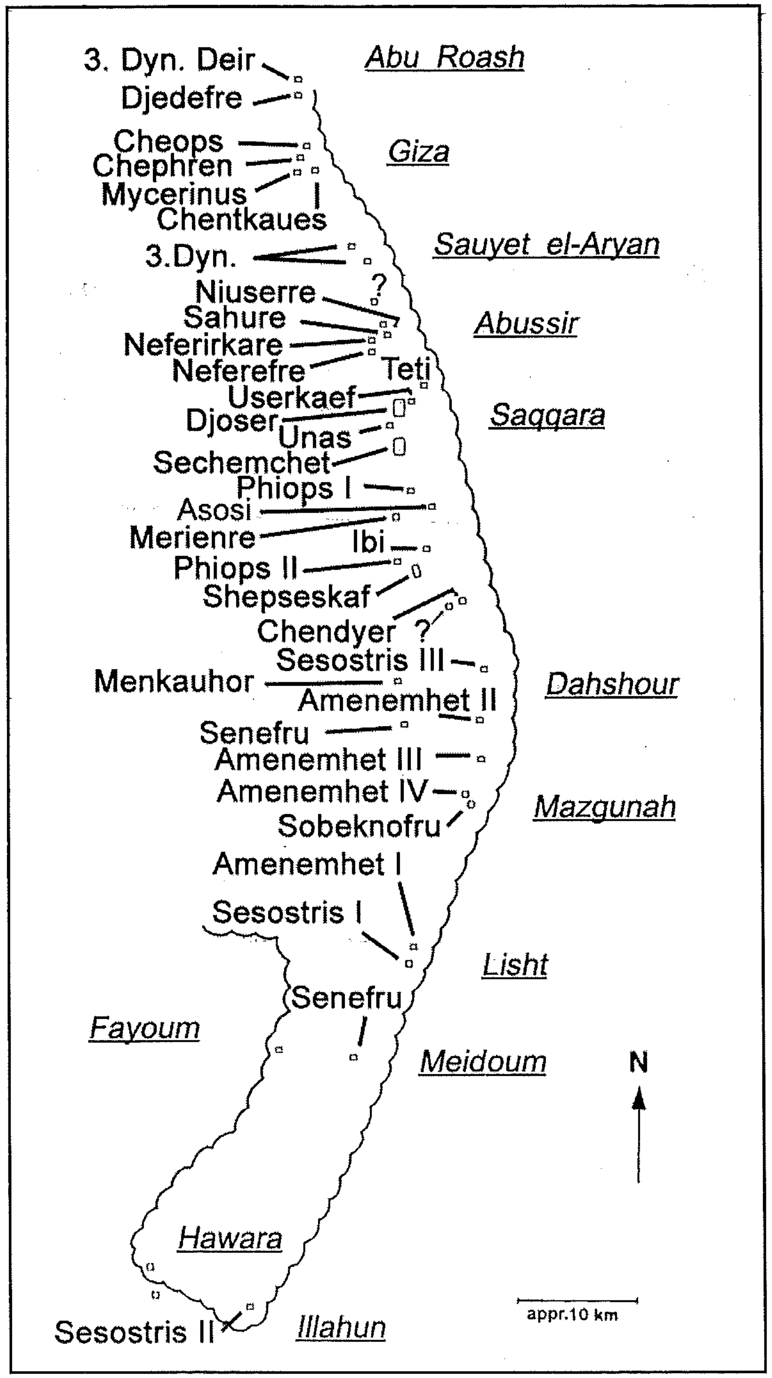

Figure 1. Map of the Pyramids [based on Helck and Otto (1982)].

1911). The deceased was put in the burial chamber with all the necessary things (food, utensils, etc) for his afterlife. Offerings were presented to him during special ceremonies, which were taking place in a funeral chapel close to the burial chamber. Thus, the tomb was divided in two rooms, the burial chamber and funeral chapel, and since earliest era it had always north-south orientation. Both rooms were underground, while a flat stone on the ground indicated the position of the tomb (Edwards 1993[9], 19-33; Emery 1962).

In the next phase of evolution a hill of sand or mud-bricks was erected above the tomb where the stone with the name of the deceased was erected, and the entire construction was surrounded by a wall. The shape of the hill was oblong and the main facade was at its east side, where a funerary stela was placed. In order to let the soul of the deceased visit the body, they gave to this stela the form of a false-door always at the east side and with the name of the deceased on it. The side walls were made of mud-bricks and in order to reduce the side pressure caused by the material inside it, all sides of the construction were made inclined. This type of tomb was named 'mastaba' after the benches by the doors of today's villages of Egypt, which are made of the same material (Edwards 1993[9], 19-33).

The burial monument was formed of an oblong mastaba, orientated from north to south, and a yard. The burial and statue room — called 'serdab' — were under the mastaba. In front of the mastaba, beside the false door at the east side there was a place where they used to worship the deceased by offerings. This worship place was at first in the open air, but later it was covered and divided into a great number of rooms. The royal tombs followed the same model. All innovations and evolutions started by the kings, who always wanted their tombs to be distinguished. They made their tombs bigger than usual to show their grandeur, and they surrounded them by a wall to cover their entrance. Gradually the height of the wall was reduced, so that it could cover only the entrance of the burial chamber and not the entire height of the construction. The result of this innovation was the two steps mastaba. A characteristic example of this form is the step mastaba/pyramid of Djoser (c. 2700 BC) at Saqqara with an oblong instead of square base. It was built in five phases: it started as a square base mastaba, then as two-step mastaba, an oblong mastaba, moving gradually to a four step pyramid and, finally, to a six-step pyramid (Helck- Otto 1982)

In Meidoum, near Fayoum, a seven-step pyramid was covered by an eight-step pyramid and later by a real pyramid (Maragioglio-Rinaldi 1964). This pyramid is attributed by some Egyptologists to Choni, the last Pharaoh of the Third Dynasty, while by some others to Snefru (Fourth Dynasty). Snefru built two pyramids in Dahshour. What is characteristic for the southern pyramid - the so-called Bent Pyramid - is that the angle of slope of its lower part is greater than that of the upper part The northern one — the Red Pyramid — is the earliest known to have been designed and executed as a true pyramid (Edwards 1993[9], 71-97). According to the spell 523 of the Pyramid Texts, the shape of the pyramid is connected with the sun worship: "May the sky make the sunlight strong for you, may you rise up to the sky as the Eye of Re, may you stand at that left Eye of Horus by means of which the speech of the gods is heard" (Seth 1908-1922, §1231; Faulkner 19852, 196). Thus, according to Edwards, the step pyramid and the pyramid were the means for the Pharaoh to cross the distance between the earth and the sky (Edwards 1993[9], 283). The Pharaohs of the Fourth Dynasty – Cheops, Chephren and Mycerinus (c. 2620-2500 BC) – built their pyramids in the necropolis of Giza (Edwards 1993[9], 98-151). The next pyramids were built in Abussir - not far from Giza – by the Pharaohs of Fifth and Sixth Dynasties (c. 2500-2200 BC) (Edwards 1993[9], 152-94).

The pyramids of ancient Egypt, which were built to ensure the eternal life of the Pharaohs, are in their place more than 4500 years. Some of them were damaged or destroyed in the course of time. But even this damage brought to light new knowledge for the science: how the pyramid was constructed in graded coats, the interesting art of joining the stones of the coating, also the system of draining away the water of the rain falling on the pyramids and the temple through special gutters and then through watercourses on the ground and the underground sewage system (Edwards 1993[9]; Borchardt 1911).

THE ACCURACY IN THE CONSTRUCTION OF PYRAMIDS

Particularly remarkable and characteristic features of the construction of the Giza pyramids are the accuracy in how they were built. In the right angles of the base of Cheops pyramid appear very small errors (Borchardt 1926):

in	NW	corner	- 0′ 2″
in	NE	"	+3′ 2″
in	SE	corner	- 3′ 33″
in	SW	"	+0′ 33″

This accuracy in construction of right angles is an obvious indication of advanced geometrical knowledge around 2600 BC.

The orientation of Cheops pyramid is astonishing not only for its accuracy but also for the method used to construct it. The NS direction differs only 2′ 30″ at its west side, 5′ 30″ at its east side from the real North and the EW direction differs 1′ 57″ at its south side and 2′ 28″ at its north side from the real West (Borchardt 1926). One method of reaching such accurate results was through the observation of sunrise or sunset during the equinox: the shade of the plumb line on the horizontal level at the moment of sunrise or sunset gives the EW direction. According to Borchardt (1926), however, the ancient Egyptians were not able to use the above way because of the following:
1) They could make this observation only twice a year. 2) According to our knowledge, the Egyptians were not able to estimate the exact days of equinox at the era of construction of the pyramids. 3) The above method of observing the sunrise and sunset would give an approximate error of 15′, which is greater than the existing errors, because of the quick transposition of the sun on the horizon at the equinoxes, so that the position of the sun is rarely at spring or fall point at the moment of sunrise or sunset. Similarly, if they had estimated the directions of both points of sunrise and sunset at the equinox and then bisected the angle of these directions, so that the bisector could give the SN direction, then the error would have been much greater.

Another way is the determination of the SN direction using the stars. At that time in the place of our Polar star, there was α Draconis, a star of magnitude 3.8, whose angular distance of North pole was 1° 15′. If α Draconis had been used, then the error would have been around 1° 15′. The position of the north could also have been found as middle of the segment, which is determined by the two positions of a star with a difference of 12 hours, which was possible only at the New Kingdom period. It might have been found also as middle of the segment determined by the position of upper and lower culmination of a star, but prerequisite for this is a special instrument for the measuring the height of a star, not known at that time too. Then the above methods are out of question.

The way that can give the accurate NS direction is the determination of the points of rising and setting of a star that remains for a short time under the horizon. The bisector of the angle ROT in Fig. 2 gives the NS direction (R: rising point, O: observer, T: setting point). Borchardt claims that this was the way used by the ancient Egyptians for the determination of the North and he mentions about harpedonaptae and the rite of stretching the cord. Harpedonaptae or "rope stretchers" were surveyors, who also had a function to perform in the laying of foundation stones for temples. In the temple of Amada, south of Aswan, there is a depiction of the rite of stretching the cord at the time of Tuthmosis III (c. 1520 BC) (Borchard 1920).

We cannot answer definitely how they were able to construct right angles or to find correctly the orientation. But these are problems, which cannot be solved without advanced geometrical and astronomical knowledge. About the competence of ancient Egyptians in Geometry, the Greek Mathematician Democritus (c. 460-370 BC) mentions that "in the construction of lines with proofs I am not surpassed, not even by the so-called harpedonaptae of the Egyptians." The comparison with Egyptians was made to show his expertness.

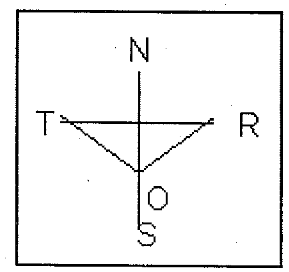

Figure 2. Determination of NS direction.

ASSOCIATED MATHEMATICS WITH PYRAMIDS CONSTRUCTION

The pyramids of the Old Kingdom were constructed between the Fourth and Sixth Dynasties (c. 2625-2200 BC). Unfortunately there are no written calculations found about Pyramids construction. The written evidences that exist at our disposal originate from papyri about a thousand years later. These are Rhind Mathematical Papyrus [RMP] (RMP 1898 / Peet 1923) and Moscow Mathematical Papyrus [MMP] (Struve 1930). Rhind Mathematical Papyrus was a teaching book, its aim being the transmission of mathematical knowledge to the future generations. What is preserved to us is a copy dated from Hyksos period of an older original Papyrus that goes back to the Middle Kingdom, as it is mentioned in the title of the Papyrus: "Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets. This book was copied in the year 33, in the 4th month of the inundation season, under the Majesty of the king of Upper and Lower Egypt, ʿA-user-Reʿ, endowed with life, in likeness to writing of old made in the time of the king of Upper and Lower Egypt, Ne-ma-ʿet-Reʿ (Amenemhet I). It is the scribe Amose who copies this writing" (Chace *et al.* 1927).

At the time of the Twelfth Dynasty (Middle Kingdom) a new fashion pushed the Pharaohs Amenemhet and Sesostris I,

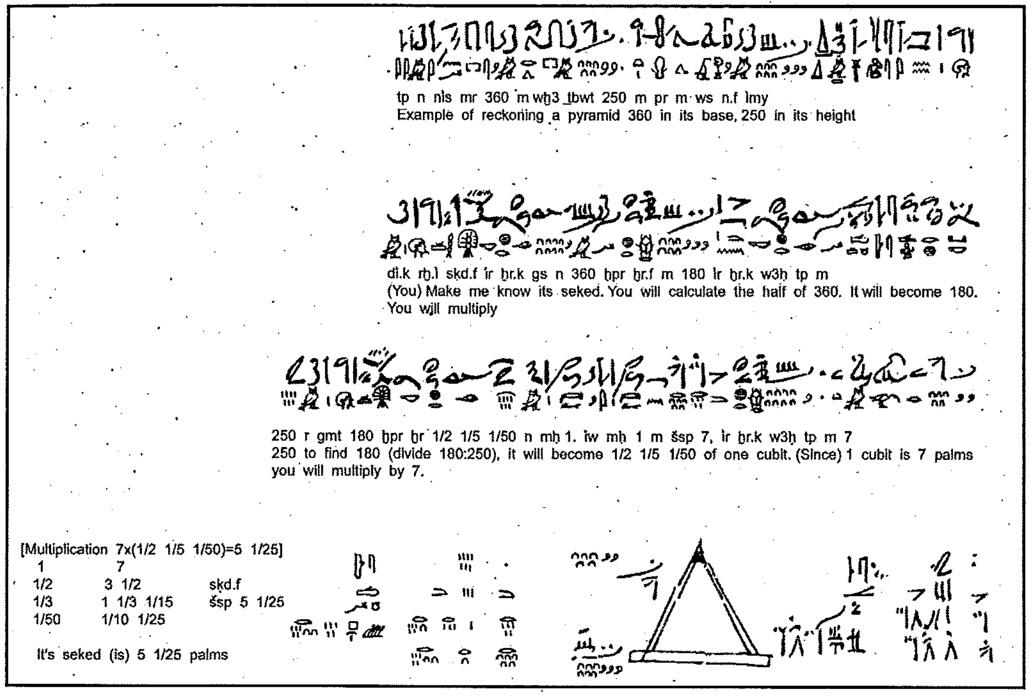

Figure 4. Facsimile of RMP 56 with transcription in hieroglyphics, transliteration and translation.

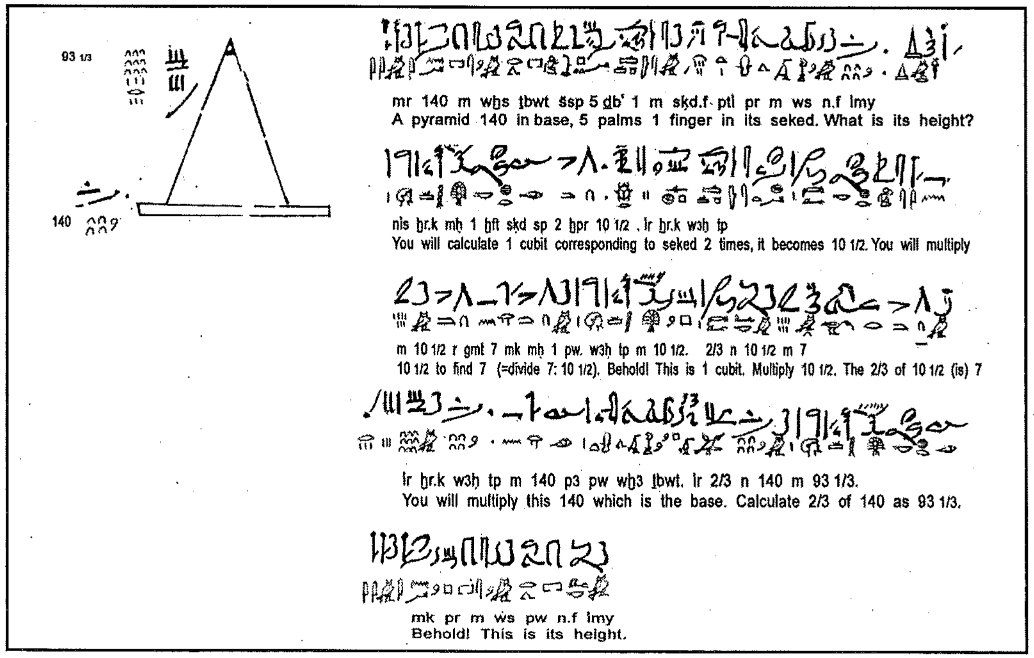

Figure 5. Facsimile of RMP 57 with transcription in hieroglyphics, transliteration and translation.

II and III to build pyramids in Dahshur, Lisht, Hawara and Illahun. From this era come both RMP and MMP. We present below the mathematical problems, concerned with the pyramids, from RMP, MMP and Demotic Papyri.

1. The 'seked' of a pyramid.

In Rhind Mathematical Papyrus [RMP 56-60] there are some problems dealing with the 'seked' (*sqd*) of a pyramid (*mr*) and the *jwn* (cone, prism, pillar).

The word 'seked' is related to the causative form of the verb *qd* 'build' (Faulkner 1991[7], 250) and denotes the slope of the inclined faces of a pyramid in such a way that reminds us of the cotangent of an angle. We take a cross section of a pyramid through the apex and the middle of each of two opposite sides of the square base (see Fig. 3). If half of the pyramid side is b palms and its height is h cubits, then the 'seked' is the quotient b palms/ h (cubits). Cubit and palm are units of length: one cubit (mh) is seven palms and one palm (*3p*) is four fingers (*db*[c]). Then the 'seked' denotes that the slope is b/h palms horizontally for every rise of one cubit in height. The differences between 'seked' and cotangent (cotx) are the following:

1) For the calculation of cotangent, both length and height is measured in the same unit, but for the calculation of 'seked' the length is always calculated in palms and the height in cubits.

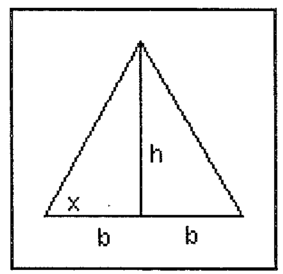

Figure 3. Cross section of the Pyramid for the definition of 'seked'.

2). The cotangent is a pure number, while the 'seked' is always expressed in palms or fractions of palms.

The existence of the notion of 'seked' was essential to build a pyramid, in order to keep the same slope for each inclined face. Every mason was able to attain the given slope of the pyramid, raising the height one cubit for every 'seked' of length horizontally. Since the ancient Egyptians could use only unit fractions, the use of palms to measure the length, instead of cubits, was providing more accurate measuring. For example, if the 'seked' 5 1/25 mentioned in RMP 56, had been calculated in cubit/cubit it would have given 1/2+1/5+1/50, which would have been difficult to realise accurately.

Problems RMP 56-60, with their working out in modern notation, are as follows:

Problem 56: "Knowing the side of the base and the height of the pyramid, find the 'seked.' Side of the base, given as *wx3 ṯbwt* (lit. "seek the sandal"): 2b=360 cubits then b=180 cubits. Height, given as *pr m ws* (lit. "go up from the crack"): h=250 cubits.

Working out:

b/h =180/250 =1/2+1/5+1/50 then seked=(1/2+1/5+1/50) .7 palms =5 1/25 palms.

Problem 57: "Knowing the side of the base and the seked, find the height."

Side of the base: 2b=140 cubits then b=70 cubits. 'Seked' =5 1/4 palms.

2 x seked=10 ½ palms. Working out:

$$\frac{h}{140\ cubits} = \frac{1\ cubit}{10\ 1/2\ \text{palms}}$$ since 1 cubit is 7 palms:

then $$\frac{1\ cubit}{10\ 1/2\ \text{palms}} = \frac{7\ palms}{10\ 1/2\ \text{palms}} = \frac{2}{3}$$

Kol. XXVII	Col. XXVII	
1. Form der Berechnung eines Pyramidenstumpfes	Example of calculation of a truncated pyramid	
2. Wenn man dir nennt einen Pyramidenstumpf von 6 (Ellen) von der Hφhe	If one tells you, a truncated pyramid 6 (cubits) in its height	h=6
3. zu 4(Ellen) auf der Unterseite, zu 2 auf der Oberseite	of 4(cubits) in lower side, of 2 in upper side	a=4,b=2
4. Rechne du mit dieser 4,quadri ert. als Voróbergehendem). Es entstehet 16.	Calculate this 4, squared, it becomes 16	$a^2=16$
5. Verdoppelte du 4. Es entstehet 8.	Double 4. It becomes 8.	a.b=8
6. Rechne du mit dieser 2,quadri ert. Es entstehet 4.	Calculate this 2 squared, it becomes 4	$b^2=4$
Kol.XXVIII	**Col. XXVII**	
1. Addiere du zusammen diese 16	Add together the 16	
2. mit dieser 8 und dieser 4	with this 8 and this 4	$a^2+a.b+b^2=$
3. Es entstehet 28. Berechne du	it becomes 28. Calculate	$=16+8+4=28$
4. 1/3 von 6. Es entstehet 2. Rechne	1/3 of 6, it becomes 2. Calculate 28 two times.	$h/3=2$
5. du mit 28 2mal. Es entstehet 56.	It becomes 56.	$V=(a^2+a.b+$
6. Siehe! es ist 56. Du hast richtig gefunden	Look! It is 56.You have found it right.	$+b^2)h/3=56$

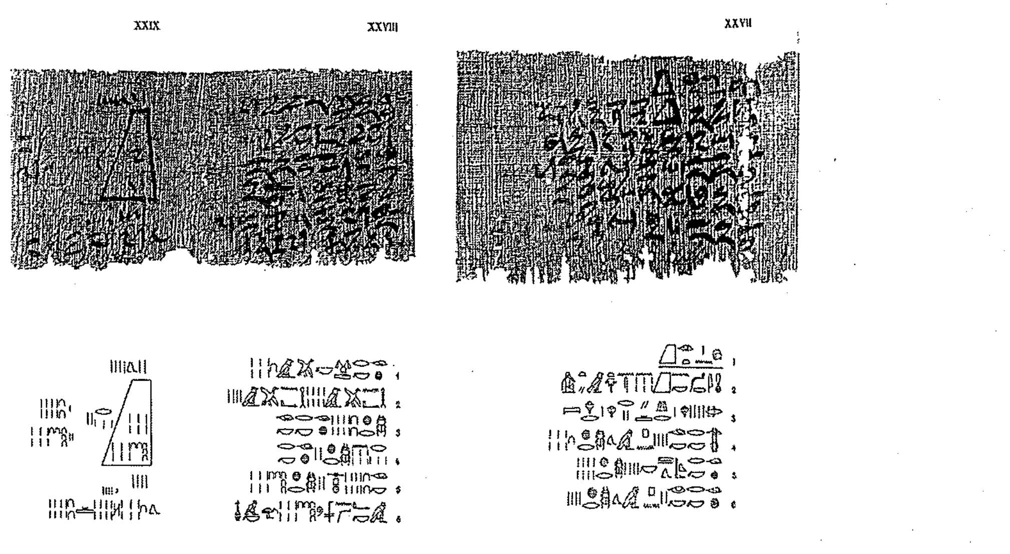

Figure 6. Facsimile and transcription in hieroglyphics of MMP 14 [taken from (Struve, 1930)].

$$\frac{h}{140\ cubits} = \frac{2}{3}$$ and h=140 cubits. 2/3=93 1/3 cubits.

Problem 58: "Knowing the side of the base and the height, find the seked."
Side of the base:2b=140 cubits then b=70 cubits. Height: h=93 1/3 cubits
Working out: Seked:

$$\frac{b}{h} = \frac{70}{93\ 1/3} = \frac{1}{2} + \frac{1}{4}$$

then seked=(1/2+ 1/4).7 palms = 5 1/4 palms.

Problem 59A: "Knowing the height and the side of the base, find the seked."
In this problem the lengths of side of the base and height have been taken vice versa. The side of the base is given 8 instead of 12 [cubits], and the height 12 instead of 8 [cubits]. 'Seked': b/h=6/8=1/2 +1/4, then 'seked' is (1/2 +1/4).7 palms =5 1/4 palms.

Problem 59B: "Knowing the side of the base and the seked, find the height."
Side of the base: 2b=12 cubits then b=6 cubits. 'Seked': 5 palms 1 finger (=5 1/4 palms). Working out: 2 x seked =10 ½ palms

$$\frac{h}{12} = \frac{1\ cubit}{10\ 1/2\ palms}$$

Since 1 cubit has 7 palms

$$\frac{1\ cubit}{10\ 1/2\ palms} = \frac{7\ cubit}{10\ 1/2\ palms} = \frac{2}{3}$$

then h/12 =2/3 and h=12 cubits. 2/3 =8 cubits (written as 4 in the text).

Notice that the problem RMP 58 is a converse of RMP 57, and RMP 59A is a converse of RMP 59B. In the latter pair there is only one figure for both problems.

Problem 60: "Knowing the base and the height of a *jwn* (cone, prism, pillar), find the seked."

Tables 1: The angles of slope of the inclined faces of some pyramids.

Cheop	51° 50′
Chephren	53° 10′
Mycerinus	51° 10′ 30′
Dahshur (southern stone pyr.)	54° 31′ (lower part) 43° 21′ (upper part)
Dahshur (northern stone pyr.)	43° 40′
Meidum	coatings 75° casing 51° 52′

Table 2: The slopes of the pyramids (*jwn*) studied in RMP.

	seked	a=cotx	x=arccota (angle of slope)	the slope is close to the pyramid of
Problem 56	5 1/25 palms	18/25	54° 14′ 46″	Dahshur (south. pyr.lower part)
Problems57-59	5 1/4 palms *stwty*	3/4 a=tanx	53° 7′ 48.37″ x=arctana	Chephren
Problem 60	4	4	75° 57′ 49.52″	inside coatings of Meidoum Pyramid

Base: 2b=15 cubits then b=7 1/2 cubits (if we take the word jwn as 'cone', then *sntt* = 2b is the diameter of its base. Height: h=30, given as *q3w n hrw* =height of the top. Working out: Instead of the 'seked', which is known from the previous problems, here a simple division h:b=30: 7 1/2= 4 gives the *stwty* ( ). This *stwty* may be the counterpart of our tangent.

The slopes of the pyramids mentioned in the above problems are not imaginary, but they come from existing buildings, as the tables 1 & 2 show (Helck-Otto 1982).

2. *The volume of a truncated pyramid.*

Blocks of stone in the shape of a truncated pyramid were used as corner stones for building pyramids. These truncated pyramids had two of their faces vertical to their bases. The Problem 14 (Fig. 6) of the Moscow Mathematical Papyrus [MMP] (Struve 1930) is concerned with the calculation of the volume of such a truncated pyramid with square bases of sides 4 and 2 respectively and height 6 (translation after Struve 1930).

The method followed in this problem can be described by the formula $V=(a^2+a.b+b^2).h/3$, where a, b are the sides of the two bases and h the height, and it is absolutely right and astonishing for that era. The corresponding formula used by the Babylonians (Van der Waerden 1954; Neugebauer 1935) in British Museum tablets 58194, 85196, 85210 was V=(a.a+b.b).h/2. This formula appears also in Brachmagupta (Colebrooke 1817; Rangacarya 1912) and in Stereometrica (ii 17, 59) of Heron of Alexandria (c. 1st - 2nd century AD), as an approximate formula (Vogel 1930).

In Greek Mathematics, Democritus firstly stated a formula for the volume of a pyramid and Eudoxus proved it (CUF 1971). Euclid in book XII, §7 of his Elements gives a proof for the volume of a pyramid, by means of resolution of a prism into three equivalent pyramids (Stamatis 1975). Heron of Alexandria in Metrica (ii 7) calculates the volume of a truncated pyramid as a difference of two pyramids, without concluding a special formula for this. The height of the small pyramid is worked out by means of a proportion. But in Metrica (ii 8) the volume of a 'vomiskos', which is a solid related to a truncated pyramid with rectangular but not similar base and top, is calculated. This solid is broken up by means of sections parallel to two of the side faces into four solids, and its volume is correctly given by the formula:

$$V=\left[\frac{1}{2}(i+m)\cdot\frac{1}{2}(j+n)+\frac{1}{2}(i-m)\cdot\frac{1}{2}(j-n)\cdot\frac{1}{3}\right]\cdot h$$

where i, j and m, n are the sides of the two rectangles and h the height. If i=j=a and m=n=b then we have a truncated pyramid like the one in MMP 14 and the formula is transformed to:

$$V=\left\{\left[\frac{1}{2}(a+b)\right]^2+\left[\frac{1}{2}(a-b)\right]^2\cdot\frac{1}{3}\right\}\cdot h=$$

$$\frac{1}{4}\left[(a+b)^2+(a-b)^2\cdot\frac{1}{3}\right]\cdot h=$$

$$=\frac{h}{4}\cdot\left(a^2+2ab+b^2+\frac{a^2}{3}-\frac{2}{3}\cdot ab+\frac{b^2}{3}\right)=$$

$$=\frac{h}{4}\cdot\left(\frac{4a^2}{3}-\frac{4}{3}\cdot ab+\frac{4b^2}{3}\right)=$$

$$=\frac{1}{3}\cdot h\cdot(a^2+ab+b^2)$$

that is exactly the formula used in MMP 14.

The use of the formula for volume of a

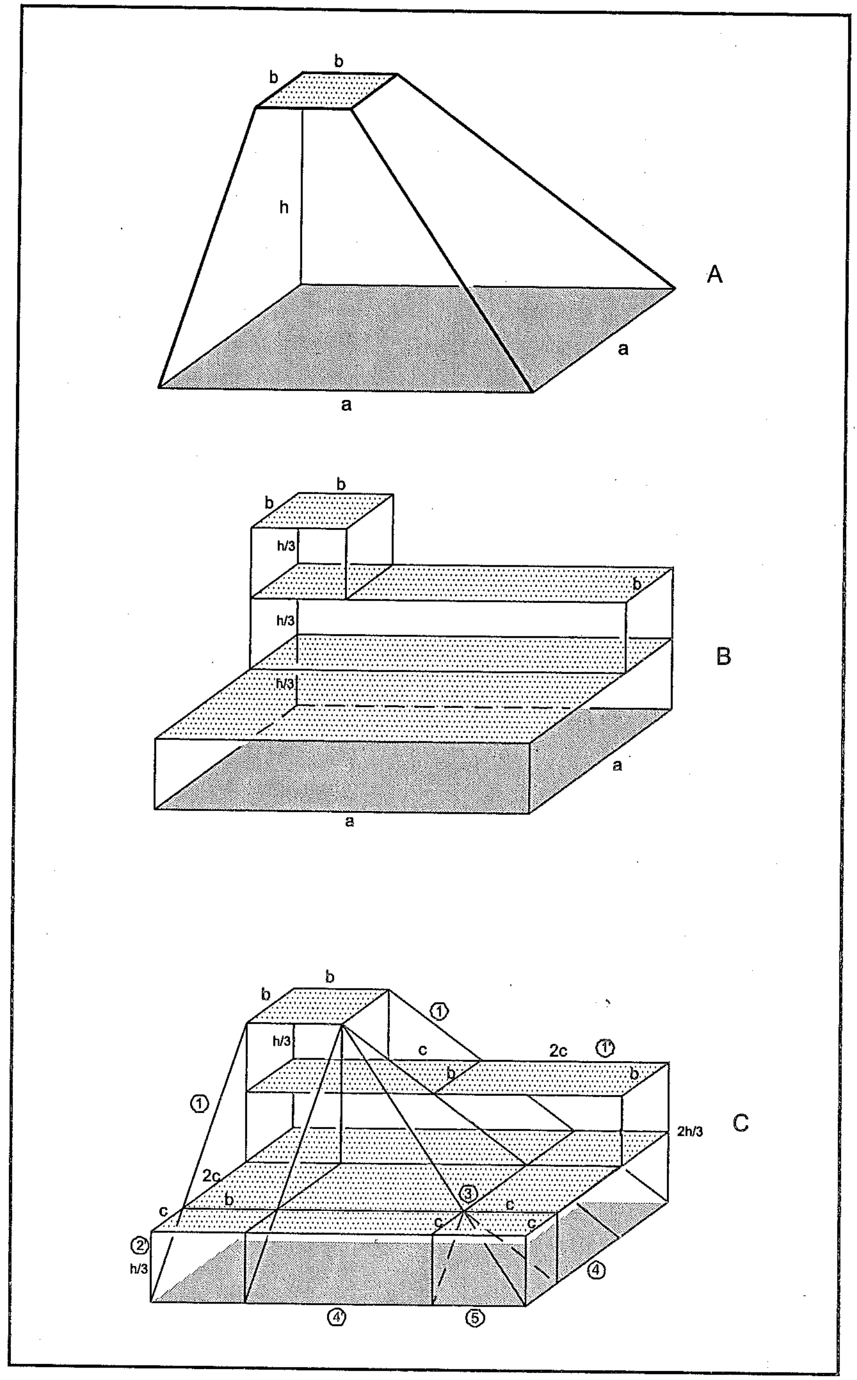

Figure 7. Transformation of a truncated pyramid into a set of 3 parallelepipeds.

truncated pyramid, implies that the ancient Egyptians were able to calculate the volume of a whole pyramid as $V=a^2.h/3$ (where a is the side of the square base and h the height of the pyramid), although there is not such a problem in the preserved Hieratic Papyri. The calculation of the volume of a pyramid appears first – among the preserved ancient Egyptian mathematical documents – in the Demotic Papyrus Cairo (problem 39), coming from the Ptolemaic Period.

The researchers of ancient Egyptian Mathematics have proposed different ways to explain how it was possible for the ancient Egyptians to deduce such a difficult formula (Gunn- Peet 1929; Vogel 1930; Luckey 1930; Thomas 1931; Vetter 1933; Cassina 1942; Gillings 1982). I would like to present my own explanation of how, by using geometrical transformations, the ancient Egyptians might have reached the formula

$$V=(a^2+ab+b^2)\cdot\frac{h}{3}\cdot$$

Fig. 7A shows a truncated pyramid with square bases, with sides of lengths a, b respectively and height h. It is supposed, that two of the faces are perpendicular to the bases and the other two are inclined. We trisect the height and we consider three parallelepipeds (Fig. 7B):

i) The first has the same square base as the pyramid and height h/3. It is situated at the bottom of the pyramid.

ii) The second has length a, breadth b and height h/3 and is situated upon the first.

iii) The third has the same square base with the top of the truncated pyramid and is situated under its top.

We will compare the solids of Figures 7A and 7B (Fig. 7C): There are 3 pairs of equal prisms: Prisms (1) and (1′) of dimensions 2c x 2h/3 x b, prisms (2) and (2′) of dimensions c x h/3 x b, prisms (4) and (4′) of dimensions c x h/3 x (a-b-c) = c x h/3 x2c. Since we trisect the height of the truncated pyramid, the dimension c is equal to (a-b)/3 because 3c=a-b. We remove from the initial truncated pyramid the prism (1) and add (1′). Similarly we remove prism (2) and add (2′). Then we remove the pyramid (3) and add prisms (4), (4′) and two of the three pyramids included in the parallelepiped (5). The volume of pyramid (3) is $V(3)=1/3.(2c)^2.2h/3=8c^2.h/9$, the volume of prism (4) is $V(4)=c/2\ .h/3.\ 2c=c^2.h/3$. The volume of the two pyramids in prism (5) is $V(5')=2.1/3.c^2.h/3=2c^2.h/9$.

The sum $V(4)+V(4')+V(5')=c^2.h/3+ +c^2.h/3 \ +2c^2.h/9=(2/3 \ +2/9).c^2.h= =8c^2.h/9$ is equal to V(3). Then the truncated pyramid is transformed into three parallelepipeds, and its volume is clearly $V=a^2.h/3 \ + \ ab.h/3 \ +b^2.h/3= =(a^2+ab+b^2).h/3$.

3. Demotic Papyri

The study of the pyramids did not stop, of course, in 1600 BC. Since there is no mathematical papyrus found from the

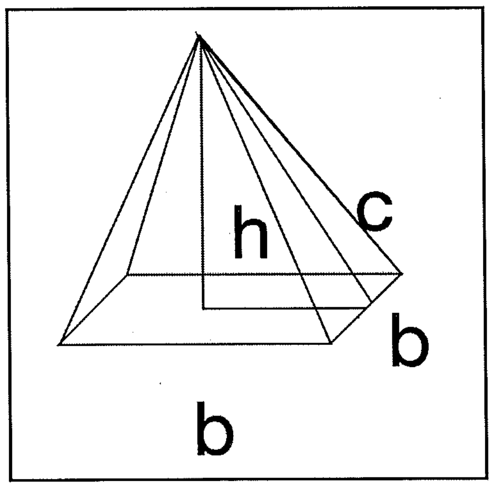

Figure 8. The Pyramid in Demotic Papyri.

period 1600-300 BC, we cannot conclude what happened in the interim. Next mathematical papyri come from the Ptolemaic period and they are written in Demotic.

In the Cairo Museum Papyri 89127-30, 89137-43, there are two problems concerned with pyramids having square bases (Parker 1972) (see Fig. 7). In problem 39, knowing the vertical height of the pyramid (h=300) and the side of the square base (b=500) the height c of a triangular face of the pyramid is calculated by the use of the formula $c^2 = h^2 + (b/2)^2$. In problem 40, knowing that both the side d of the base and the height h of the pyramid are 10 cubits, the volume of the pyramid in cubic cubits is calculated as $V=(b\cdot b\cdot h)/3$ (Fig. 8).

CONCLUSION

Since the Egyptians decided to construct pyramids they had to study how to do it. The necessary accuracy for the construction needed to be based on mathematical and astronomical knowledge, and the result demonstrates us that this was present. The accuracy of Pyramid orientation based on astronomical observations, combined with other evidences as the 365 days' year, already established at the time of Old Kingdom (Clagett 1995), which was divided into 12 months of 30 days, plus 5 epagomenal days, and adopted by the astronomers of the Hellenistic period and the measuring of time during night by the use of certain stars, reveals the high – for that era – level of astronomy and the accuracy of astronomical observations.

The preserved Mathematical Papyri manifest the creditable performance of Egyptians in accuracy: very good approximations of the area of the circle, the hemisphere and the volume of cylinder, complicated equations correctly solve, correct formulas for arithmetic progressions etc. (Vafea 1998). In Mathematics, concerned with Pyramids construction, few but indicative cases are preserved. They had to calculate the volume of a truncated pyramid and they did it successfully. They had to find an easy way for the masons to attain the given slope of the pyramid, so they invented the notions of 'seked' and *stwty*. In that sense, they were the first to use Trigonometry, expressing the size of the acute angles of a right-angled triangle as ratio of its two perpendicular sides. The use of notion of 'seked' implies also the knowledge that similar triangles have their corresponding sides in proportion. The etymology of the word 'seked' and the mention of 'seked' of existing pyramids in RMP prove, that these problems have their roots at the time of pyramids construction. Although mathematical documents from the period between 1600-300 BC are lacking, the appearance of problems concerning pyramids in Demotic writing, keeping the style of Middle Kingdom mathematical papyri even in the Hellenistic Era, indicates the persistence of the Egyptians in the study of pyramids and, thus, the importance of the latter. The few written documents accompanied by the study of existing Pyramids can assure us of the flourishing of Mathematics in ancient Egypt.

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