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ASTRONOMICAL RECKONING OF THE GREAT PYRAMID'S ENTRANCE TILT, USING THE 2/n TABLE, THE SINE CALCULATION AND THE GRID SYSTEM FROM RHIND MATHEMATICAL PAPYRUS

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ABSTRACT

The paper explains how the ancient Egyptian architects used the arithmetic of unit-fractions to reckon the astronomical tilt of 26° 31' 23" ±5" for the Great Pyramid's entrance passage, which is correlated to the latitude of ~ 29° 58' 45" north and Earth's axial tilt ~24° 6' of nearly 5070 years ago. Therefore, in the beginning, the paper explains for the first time, with the supporting translation, the architectonic and geometric reckoning-methods of almost 40% of the so-called Rhind Mathematical Papyrus (RMP). Firstly, it explains the meanings of 36 mathematical and geometric symbols in RMP's hieratic text. Secondly, in relation to the divisions of the Egyptian cubit rod, it explains the architectonic decomposition method of the $2/n$ table on decomposing the sum of two unit-fractions, such as $1/n$, into other unit fractions, where n is an odd number from 3 to 101. It shows that this recto table, which represents almost 1/3 of RMP, is on subdividing line-segments like $2/n$ into only measurable parts. Thirdly, it shows that RMP#24 is an example of calculations related to the sine of an angle and RMP#74 is an example of calculating the values of angles. Fourthly, it shows that RMP#65 is an example of using a grid-system to plot and subdivide the arc of half the side of an octagon into 10 parts. Finally, the paper shows how the pyramids designers used these ancient Egyptian mathematical and geometric methods in reckoning and implementing the astronomical tilt of the Great Pyramid's entrance passage in the Giza Plateau.

KEYWORDS: Egyptian Mathematics; Rhind Papyrus; 2/n Table; RMP#24; RMP#51; RMP#65; RMP#74; Giza Pyramids; Great Pyramid; Harpedonaptae.

1. INTRODUCTION

The pyramids of the 4th Dynasty in ancient Egypt (Manetho, 1940-1964, pp.45-50) were and still are the most outstanding architectonic megalithic-works in the history of the humankinds. The term architectonic means using basic sciences in designing architectural and site-planning projects. It is closely related to the new scientific field of Archaeoastronomy that searches on the astronomical information that were used and encoded in the design of the invaluable architectural heritage of the ancient civilizations. Citing Ibn Wsif Shah and Ibn Salama Al-Quda'i, Al-Maqrizi (1364–1442AD, pp.319-346) mentioned in the pyramids chapter, about 130 Egyptian priests had participated in designing and implementing the pyramids of the 4th Dynasty; they were specialized in construction sciences, astronomy, mathematics, and the allied ancient disciplines. He also mentioned the written text on an Egyptian golden tablet that was translated to King Philip-II of Macedon. The tablet's text reveals the reason behind building the pyramids of the 4th Dynasty, based on understanding the cycle of life and extinction of our planet Earth and their knowledge about the times of the frequent and diverse cataclysmic events in each obliquity cycle of our planet (Aboulfotouh, 2007). Particularly the cycle that was named after *Hor-Mageed-Don* or Falcon of the Mighty God (Armageddon, i.e., king *Suphis-I* or Sphinx) that is almost 5070 years (Aboulfotouh, 2017). The discourse between Solon and the Egyptian priests on the outcomes of Earth's cataclysmic events (Plato, 330BC) indicates that the Ancient Egyptians were highly acquainted about this knowledge; see also (Liritzis *et al*, 2019) on the ancient cataclysmic events.

However, designing and constructing megalithic structures like the Bent-Pyramid in Dahshure and the Great Pyramid in Giza, the best astronomical-models in pyramids design theory (Aboulfotouh, 2015) requires having some technical and applied knowledge in the fields of mathematics, astronomy, and megalithic construction. On the one hand, transporting and lifting of large stone-blocks is the first thing that most scholars are wondering how they did it. Based on the written texts by the acquainted historians, e.g., (Herodotus, 484-425BC, p.427) & (Hassan, 2001, p.88), the construction of megalithic structures like the Pyramids of Egypt relies on the manufacture of strong-ropes, rigid unyielding-spools, and steady wooden-cranes; as well as knowing the characteristics of stones and the smart-use of scales. One of the supporting technical texts was found in the tomb of the so-called *Tehuti-Hetep* in El-Bersheh, which describes in detail, how had the ancient Egyptians transported a colossal statue equals the weight of 1000 men by only 172 men (Newberry *et al*, 1895, plate-XIV) & (Nosonovsky, 2007), using a suitable lubrication method (Li *et al*, 2013).

On the other hand, regarding pyramids design and the related reckonings, because the meanings of all the geometric-symbols in the hieratic math-texts are not deciphered yet (Aboulfotouh, 2019), scholars of the history of mathematics did not see strong evidence in the found math papyri related to the pyramids design-theories, other than reckoning the slope ratios of pyramids, e.g., see Griffith's opinion (Gillings, 1982, p.48). In 2007, this author (Aboulfotouh, 2007) retrieved the astronomical equations of reckoning the tilts of the entrance passages of the largest five pyramids of the 4th Dynasty, with regard to place and time, which are represented by the "latitude of the place" and the "angles of earth's obliquity range", respectively. Each equation (for each pyramid) has been formulated to encode in each tilt an array of, diverse and integrated, information about the place and/or time, based on the designer's idea about the earth's obliquity range and the main time intervals in each obliquity cycle. These equations, as well as the other astronomical algorithms of pyramids design models (Aboulfotouh, 2014 & 2015), are based on understanding the trigonometric reckonings of sine and cosine of an angle. For example, Eq.1 reckons the tilt *a* of the entrance passage of the Great Pyramid (Figure 1) in Giza plateau (Aboulfotouh, 2007):

$$\sin\alpha = \frac{\sin\lambda * \sin O_m}{\sqrt[2]{1-\frac{O_i^2}{O_t^2}}} \qquad (1)$$

Where λ is the latitude of the place, O_m and O_i are the Earth's mean and minimum obliquity angles respectively, and O_t is the encoded obliquity of time that implies the date of an important event. Eq.1 was formulated based on the idea of the "contour-circles" that correlates the radius of a circle that do expand (or do shrink) relative to a 2nd circle that has a constant radius and forms the frame of reference for the 1st circle, and where both have the same center, see (Aboulfotouh, 2007). It is like, for example, the circle that appears and expands on the surface of water after throwing a stone in the river. In Eq.1, the obliquity-of-time factor $[1-(O_i^2/O_t^2)]^{1/2}$ represents geometrically a cosine of such angle φ in a right-angle triangle, where the spread-out length of O_t (the shrinking radius in the obliquity's descending phase) is its hypotenuse and the spread-out length of O_i (fixed radius) is its subtending side, i.e., sine $\varphi = O_i/O_t$. In this regard, despite the difference in concept and application, Lorentz factor of time dilation $[1-(v^2/c^2)]^{1/2}$ (Einstein, 1921, pp.36-41) was also represented geometrically in modern research in applied physics with a graph of cosine of an angle φ, where sine $\varphi = v/c$, see (Orozovic, 2020).

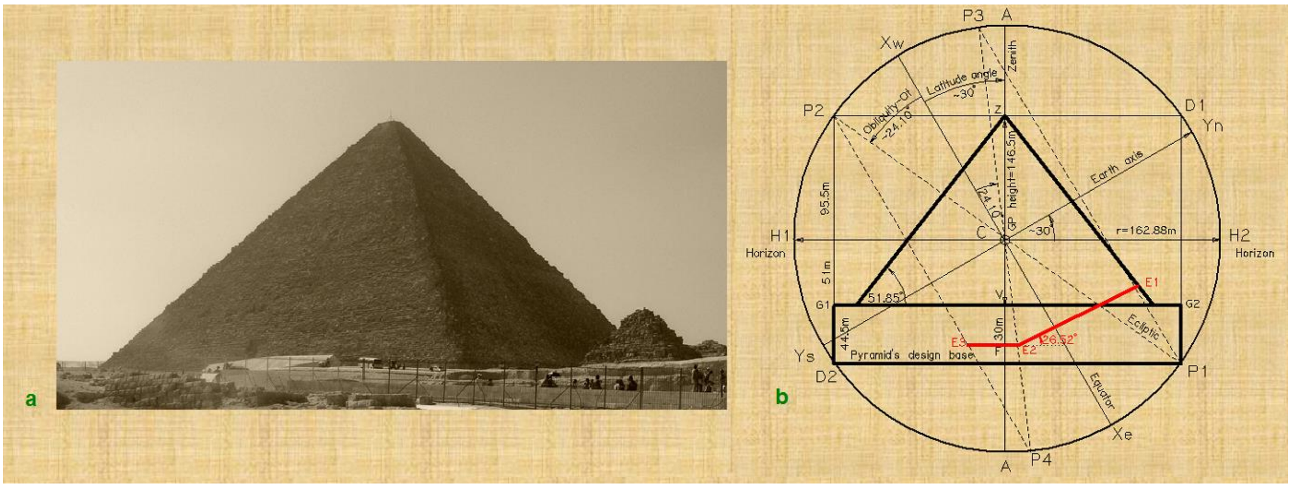

Figure 1. The Great Pyramid in Giza Plateau. a. shows a picture of the eastern and southern surfaces of the Great Pyramid (in 2015). b. shows a north-south cross section in a spherical co-ordinates system that shows the astronomical design parameters of the Great Pyramid, while looking due west; where the obliquity of time O_t=~ 24.10°, the latitude λ= ~ 30° north, and the tilt a of the entrance passage = ~ 26.52°. Besides, the entrance passage is marked by the red line E_1-E_2, and the horizontal part is E_2-E_3, where the tilt starts at point E_2 on the line P_3-P_4 (at 30m below the ground-line G_1-G_2), see (Aboulfotouh, 2015).

Moreover, the published works on the so-called Rhind Mathematical Papyrus (RMP) do show on trigonometry only the four problems from RMP#56 to RPM#59 that reckon the slopes of pyramids' surfaces and RPM#60 on a cone's slope. RMP was found in Luxor in 1858 and it is now (on display) in the British Museum (Clagett, 1999, pp.113-114). It was copied in circa 1550BC (Aboulfotouh, 2019) & (Britishmuseum.org, EA10057&58). After the table of contents and the prelude, RMP contains the $2/n$ table and 84 mathematical problems. The $2/n$ table shows the best decompositions of the sum of two unit-fractions, such as $1/n$, into other unit fractions, where n is an odd number from 3 to 101; it represents 1/3 of RMP. Figure 2 shows RMP's table of contents, its prelude, and part of the $2/n$ table from 2/3 to 2/15. The early philologists did not recognize that there are other problems in RMP (like, e.g., RMP#24) about the sine of the angle. This is in spite of the last part (in black) in the first line of the list of contents in RMP's prelude says, the papyrus includes part "on the subtending side of an angle", see Figure 2. Unfortunately, the early philologists did not decipher all types of angle symbols in RMP. Therefore, Chace *et al* (1927-1929, p.25) wrote for example, RMP#24-38 are essentially problems in divisions by fractional expression. Besides, the method of decomposition of the $2/n$ table in RMP (Gillings, 1982, p.45) never was correlated to the Egyptian cubit rod that was used in the design and implementation of buildings and in site planning. In addition, they did not decipher too the geometric symbols in the text on the survey techniques of how to subdivide and plot the diverse intervals of an arc in RMP#65, using a grid-system of rectangles. It is the survey technique that defiantly suitable for plotting the circular horizon of the Giza Pyramids, of 746m radius in the field (Aboulfotouh, 2002 & 2014). Hence, e.g., Chace *et al* (1927-1929, p.29) thought that RMP#65 is on the distribution of 100 loaves among 10 men. In this regard, this author (Aboulfotouh, 2019) showed that the first sentence in line-3 in RMP's prelude says, "Book on segments binary (the) parts (of the) unit", and the last sentence in line-3 says, "it is for the surveyors *Irrapedon*". In addition, the 1st two words in line-4 say, "to measure lands *Qeyas A'pateh*" (Fig. 2). In the Greek literatures, the professional of land-survey in ancient Egypt were called Harpedonaptae (Heath, 1921, p.121) or Harpedonaptai that was translated as rope stretchers or rope fasteners. Harpedonaptae sounds like the current Egyptian term *Harriepht A'pateh* that means "Professionals (of) Lands", respectively. It was shown too that the Harpedonaptae knew the right-angle triangle 6-8-10 (Aboulfotouh, 2019), similar to the triangle(s) of Pythagoras (Chiotis, 2021); and they used the complex fractions like (7+ 1/2)/100; see RMP#53-54 in (Aboulfotouh, 2019).

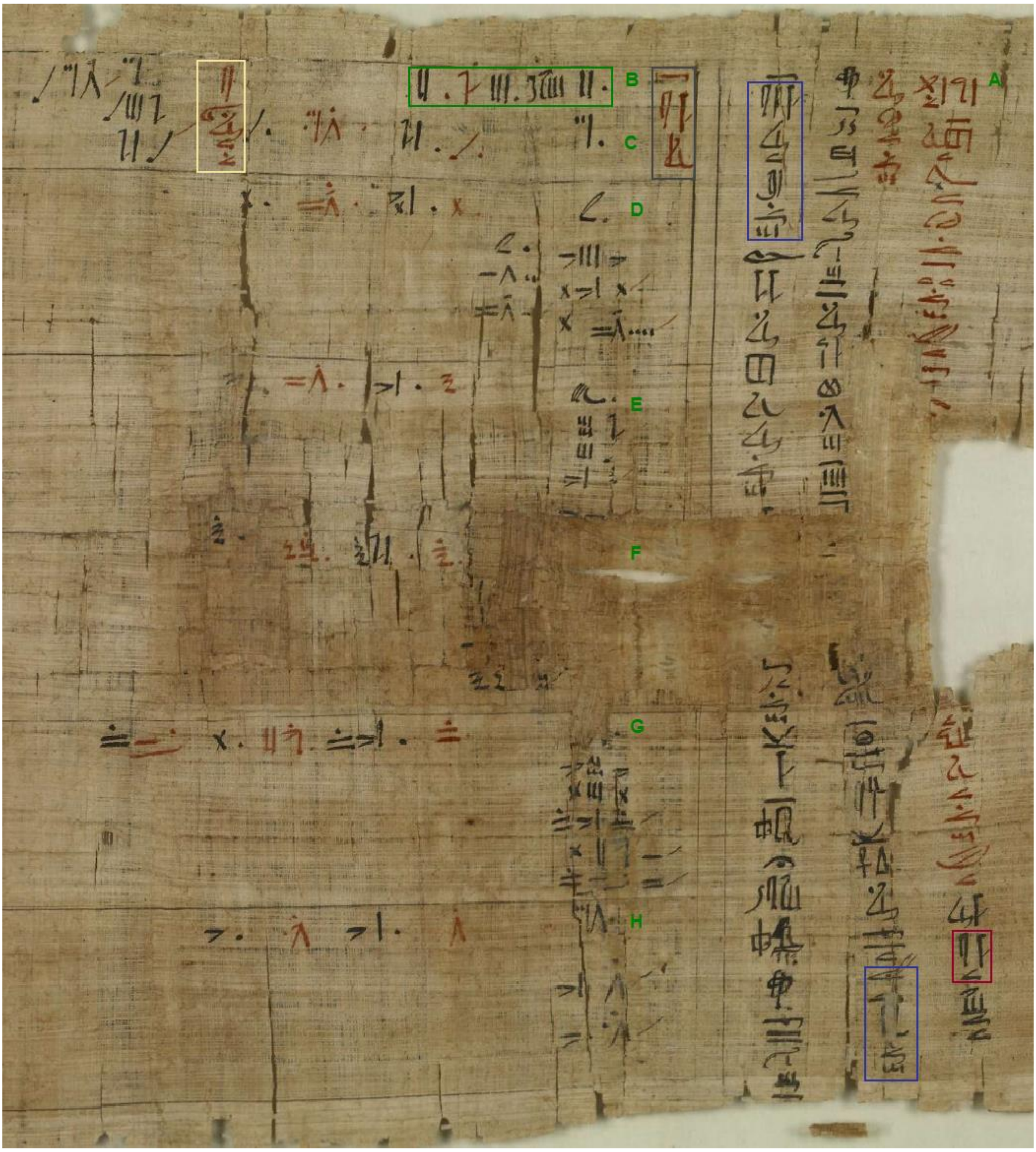

Figure 2. Part of Rhind Mathematical Papyrus (RMP). a- shows the list of contents and the prelude of RMP (right), where line-1 & line-2 are the list of contents. The words in a red box in line-1 mean the subtending side of an angle (see section-2). The words in the blue boxes (in line-3 and line-4) are "Irrabedon" and "Qeyas Phat-hat or A'pateh", which mean, "surveyors" and "measure lands", respectively. The left side shows part of the recto table on decomposing fractions like 2/n into unit fractions; where n is an odd number from 3 to 101. E.g., b is on 2/3 in a green box, c is on 2/5, d is on 2/7, e is on 2/9, f is on 2/11, g is on 2/13, and h is on 2/15. The title sentence in the gray box means "on line-S's upright length", and the words in the yellow box mean "thicknesses' total".

The claim that RMP's source was from the period of the 12th dynasty (Chace *et al*, 1927-1929, p.1) & (Clagett, 1999, p.185) has no written evidence in RMP. This is because the remaining letters in the cartouche in line-4 in its prelude (Figure 2) just says *Addiah* or *Addien,* which is the plural of folk of *Add* (Hyksos), and this implies, only, the period that during which it was copied from the original source. Besides, this author has proved that the height of each of the largest five pyramids of the 4th Dynasty is composed of 7 design-modules with a constant sub-divider length (1/2 cubit of 45cm), which conform to the pyramids problems RMP#56,57,&58 (Aboulfotouh, 2015); and this strongly prove that RMP's architectonic mathematics go back to the period of the 4th Dynasty.

Over the last fourteen decades, scholars of the history of mathematics have done great efforts in deciphering and translating the mathematical text of RMP into different languages, e.g., (Eisenlohr, 1877), (Peet, 1923), (Chace *et al*, 1927-1929), (Archibald, 1930), (Van Der Waerden, 1975), (Gillings, 1982), (Robins & Shute, 1987), (Clagett, 1999), (Michel, 2014), and (Imhausen, 2016). Besides, Archibald (1930) and Gillings (1982, p.48) have mentioned names of other scholars too. Their most important achievements were that they discovered the values of the ancient Egyptian numeral signs, and they understood the reckoning procedures of RMP's problems. But the early philologists (e.g., Moller, 1927) did not help them much to fully comprehend nearly all the geometric symbols in the hieratic math-texts; and as a result, the true written purposes of many problems in RMP were not understood. However, without their serious works, any scholar will start from scratch. Hence, building upon their works, together with using the author's architectonic background on deciphering the meaning and geometry of the Egyptian math symbols in the hieratic texts, e.g., (Aboulfotouh, 2012 & 2019), this work aims to expand slightly the frontier of the field of ancient Egyptian mathematics beyond its current domain.

Therefore, using hard evidence, this paper shows that the math knowledge in RMP is correlated to reckoning and implementing the astronomical tilt of the Great Pyramid's entrance passage. The paper is structured in five sections after the introduction (Section-1). Section-2 explains the meanings of some mathematical and geometric symbols in RMP. Section-3 explains the architectonic method of the $2/n$ table in RMP, on re-subdividing the sum of two segments from a line of n equal segments. Section-4 is on the trigonometric reckoning of the sine of an angle and the values of angles in RMP. Section-5 is on the arcs and sides of an octagon and using the grid-system in RMP. Finally, Section-6 is a discussion on reckoning and implementing the tilt of the Great Pyramid's entrance passage, using the arithmetic of unit fractions and the divisions of the ancient Egyptian cubit rod.

2. SOME BASIC MATHEMATICAL AND GEOMETRIC SYMBOLS IN RMP

Based on reviewing the hieratic text of the $2/n$ table and the 84 problems in RMP, see a copy of RMP's hieratic text in (Clagett, 1999, plates#1-105), it appears that the geometry of the ancient Egyptian Harpedonaptae primarily relies on the concept of the circle, where all the geometric problems in RMP are referenced to it, e.g., RMP#53-54&55 (Aboulfotouh, 2019). Regarding the scale, RMP#48 and RMP#50 deal with the circle that its diameter is 9 units or intervals, see e.g., (Gillings, 1982, pp.139-141) & (Clagett, 1999, plates#69&72). In RMP#53-54, these 9 units were partitioned into 10 parts (Aboulfotouh, 2019). Related to this concept, the following mathematical and geometric symbols were not decoded by the early philologists; see their early suggested meanings and transliterations in the book of Chace *et al* (1927-1929, Vol. II); they did not decipher too the real meanings of some hieroglyphic figures (signs) in the daily life.

In RMP, the circle's diameter, or part of it, as a side of such geometric shape, is symbolized by the hieratic letter *R* (horizontal plan of a vessel in hieroglyphs); and, the height or the side of the geometric shape that is perpendicular to the direction of the diameter *R*, is symbolized by the letter *S* (vertical side-view of a hoist rope), e.g., RMP#51. Besides, *R* as a symbol of motion that means to travel (or go to) in the ancient Egyptian texts was also used in combination with a line or an arc to denote a perpendicular-motion (e.g., RMP#51), an inclined-motion (e.g., RMP#56), and a curved-motion (e.g., RMP#40) of a point, or rotation around a central pillar , i.e., a perimeter (e.g., RMP#56). In RMP, *R* also implies parts or segments of a line or a module (e.g., RMP#65).

Besides, some of the geometric problems in RMP deal with both *R* and *S* together, such as RMP#51 (Figure 3a) on how to reckon the area of a right-angle triangle, see, e.g., (Gillings, 1982, p.138). In RMP#51, the triangle's base is the diameter *R* (of 9 units) that equals 10 modules, and its height *S* equals 4 modules, where each module is 1,000 units (i.e., 100*10). The ancient Egyptian method was multiplying the triangle's base *R* of 10 modules by the average height, i.e., 2 modules, at the center of the circle, as shown in Figure 3b. In RMP#51's 2nd line, we see the hieratic sign of doubling (or repeating once) that looks like number **2**. Also, for denoting a segment of length, they used the segment or delta δ sign that looks like the letter **S**, e.g., large delta and small delta, which are derived from the shape of one segment of length (e.g., RMP#65), and each curve marks a linear segment of length as in the shape of the dollar sign **$**. Regarding ratios, they used diverse types such as the symbol of the ratio between the lengths of 2 lines or 2 arcs (e.g., RMP#65). Besides, they used two types of signs to denote total: the sign of total length with an inclined stroke above the letter *t* , and the sign of part of a unit of length (or area) with a dot instead of a stroke. In RMP#40, despite the total length of one degree of arc is 100, the early philologists claimed that it deals with the quantity of loaves (Clagett, 1999, p.155). Regarding angles, there are diverse symbols in RMP such as the angle sign (e.g., line-1 in RMP's prelude & RMP#74); it is longer than the sign of 1/8 (Aboulfotouh, 2012). The coming sections include the explanation of other mathematical and geometric symbols in RMP.

Figure 3. Explaining the geometric figure in RMP#51. a- shows the hieratic text of RMP#51. b- shows the right-angle triangle EWA in RMP#51; where EW is the diameter R of a circle and the triangle's base, WA is the triangle's height S, and the average height CD is the length that repeats R in order to reckon the area of the triangle.

3. THE ARCHITECTONIC METHOD OF THE 2/n TABLE

Scholars of the history of mathematics have studied the 2/*n* table (Figure 2 & 5) in RMP, with the intention to find one equation (or an algorithm) that could yield the same shown answers of arithmetic decompositions in the table, e.g., (Gillings, 1982, pp.45-70) & (Abdulaziz, 2008). The reason of this is most likely related to viewing RMP as a book of math for basic education, and not for the technical applications by the Harpedonaptae (land-survey professionals, e.g., architects, planners, and surveyors). In RMP, the 2/*n* table starts with the title "on line-*S*'s upright-length " (Figure 2), where in the Egyptian math, the palm reed implies a side of a geometric shape, i.e., a line (e.g., line-1 in RMP's prelude & RMP#35). As the core subject of RMP is on the "binary parts (of the) unit ", this table is on re-subdividing the sum of 2 equal segments from the line-*S* (length *S*) that contains *n* equal segments. Because it is easy to find the answer in case if *n* is even number, the table shows only the cases of odd numbers. As examples for the ancient architects, planners, and land-surveyors, the table starts from *n*=3 to *n*=101. For *n*=3, the title sentence says, "the upright-length of 2 parts of the line-segment of 3"; then, the answer, (from the case of 2/5) starts with a sentence that says, "thicknesses' (*Smkt*) total , or "; since, in the case of 2/3 as a basic fraction, the ancient Egyptian Harpedonaptae used it as is without showing its decomposition.

Because RMP was written primarily for the Harpedonaptae, the method of decomposition in the 2/*n* table of the line-*S* (length *S*) is linked to the divisions of the Egyptian cubit rod. As known, the ancient Egyptians used cubits of diverse lengths, e.g., 45cm and 52.5cm (see Figure 4a), where the largest (52.5cm) is the royal cubit of 6 royal palms, i.e., spans (Herodotus, 484-425BC, p.459) that equals the length of 7 mean-palms. Each mean-palm (7.5cm) equals the length of 4 mean-fingers (each is 1.875cm), and the mean-finger's divisions are from 1/2 to 1/16 of the mean-finger, see (Aboulfotouh, 2015). Accordingly, the divisions (below 1/2) of the Egyptian royal cubit rod of 52.5cm are from 1/7 to 1/448 of the royal cubit, where 448= 4*7*16. Here, 1/448 of the royal cubit was the minimum usable measurement-unit, either in drawings or in the field. This implies that for a length of one royal fathom *οργιά* of 112 mean-fingers (4 royal cubits, i.e., 210cm), the minimum measurement-unit is 1/1792 of a royal fathom.

Moreover, in architecture practice, dividing a line into sub-segments is subject to the condition that their widths (thicknesses) could be measured by a measurement rod. As shown in Figure 4b, in order to subdivide the line *AB* into 7 equal parts, the architect can draw an auxiliary-line *BC* equals 7 mean-fingers, perpendicular to *AB* and draw the line *CA*. Then, in the right-angle triangle *ABC*, drawing lines, parallel to the hypotenuse *CA*, at the interval of each mean-finger of *CB* will subdivide *AB* into 7 equal parts. Besides, *AB* could be of any length even equal to the length of the auxiliary-line *BC*, i.e., the angle *ACB*= 45°.

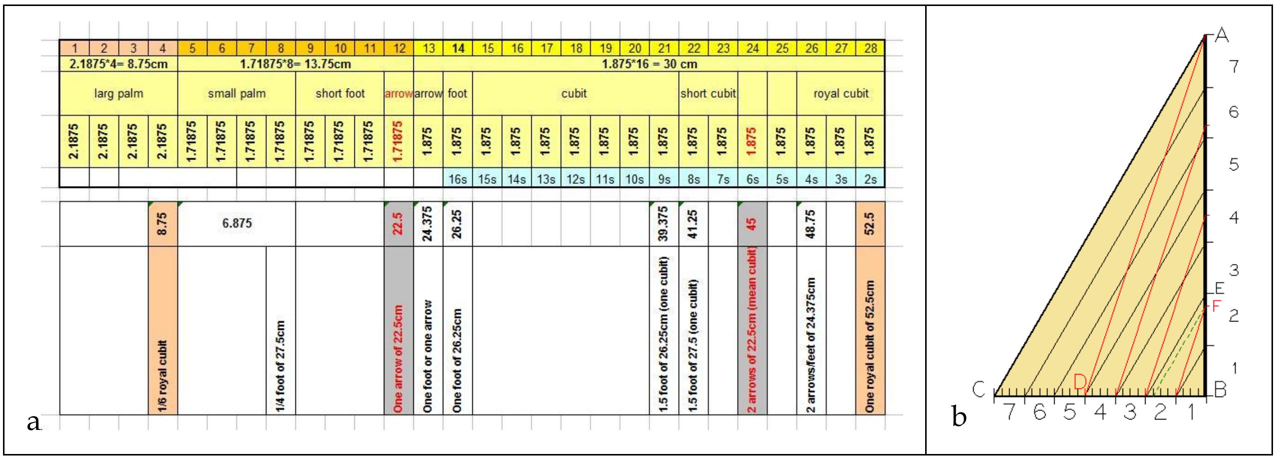

Figure 4. The Egyptian cubit rod and how to subdivide a line of any length. a- shows the design of the diverse types of Egyptian cubits in the royal cubit rod of 52.5cm (see, Aboulfotouh, 2015). b- shows how to subdivide the line AB (i.e., line-S) of any length into seven equal segments, using the auxiliary line CB of 7 mean-fingers, and re-subdividing the length BE that equals 2/7 AB (using a 2nd auxiliary line BD= 4 mean-fingers) into BF= ¼ AB, and FE= 1/28 AB.

Similarly, one can also subdivide each of the 7 mean-fingers of *CB* into smaller measurable sub-intervals from 2 to 16 segments, where the lesser the number of intervals the shorter the time the subdividing process takes. For this architectonic purpose, and for time saving, one can construct an auxiliary-table similar to that in Figure 5 to show all the possible numbers of sub-intervals (segments) in the line-*S*, e.g., for the odd numbers from n=3 to n=101, which includes 50 rows and the measurable sub-intervals in 15 columns. For example, in the row of n=7 mean-fingers, the 15 measurable sub-intervals are the results of: (2*7), (3*7), (4*7), …, and (16*7); where each represents possible measurable divisions of the auxiliary line *BC*, either in drawings or in the field. In the $2/n$ table of odd numbers n, the Harpedonaptae re-subdivided the sum of 2 mean-fingers only in the range of 3.5 royal cubits (i.e., 7 short feet or 98 mean-fingers), excluding the base 3 mean-fingers, which seem enough to do the various types of reckonings for both design and implementation.

Based on this simple notion, the core architectonic idea of the $2/n$ table in RMP (Figure 5) could be explained, using the case of the sum of two segments that each is $1/7$ of the line-*S's* length. In RMP, the segment $2/7$ of the line-*S* was re-subdivided into 2 parts: $1/4$ and $1/28$; where 28= 4*7. In practice, and based on trial and error, in order to re-subdivide any segment composed of 2 equal parts, from the line-*S* (*AB*), into other parts using a measurement rod, the denominator of the first partition m_x should be more than half n of the line-*S* (or the auxiliary-line *CB*). Hence, for the numerator 2, it starts from $(n+1)/2$; i.e., to approximate the result of $n/2$ (or n divided by any numerator) to the next upper natural number. Abdulaziz (2008) thought that $(n+1)/2$ is the last choice, which implies there were no single sequential procedure for re-subdividing all the line-segments. Hence, $(n+1)/2$ as the minimum value should be the first choice. Then, one can sequentially try other numbers above it in order to find the best architectonic answer, provided that n/m_x should yield only unit fractions plus 1, and this is the 1st proviso-i. The best answer should agree with other three provisos, where math scholars have noticed both the 2nd and 3rd provisos, as follows: (2nd proviso-ii) includes the least number of partitions as much as possible; (3rd proviso-iii) includes the lower denominator values as much as possible (Gillings, 1982, p.49); and (4th proviso-iv) the largest denominator is 16 times n.

Moreover, in all possible answers, the denominator of the 1st partition m_x (e.g., 4 for 2/7) is the number of intervals (mean-fingers) in a 2nd auxiliary line like *DB* in Figure 4b; and the denominators of the other partitions (e.g., 28 for 2/7) are possible sub-intervals in the 1st auxiliary-line *CB*, that appear in the corresponding row of 15 possibilities in the auxiliary table (Figure 5). Since *CB* may equal the line-*S* (*AB*), the number of mean-fingers in the 2nd auxiliary-line *BD* represent the alternate length *S* (alternate-*S*) for both *CB* and the line-*S*. For n=7 mean-fingers, the length of *BD* as alternate-*S* equals 4 mean-fingers, which equals the minimum value $(n+1)/2$ and agrees with the four provisos that are mentioned herein above. Whereas in this answer, the sum of 2 segments from the line-*S* of 7 equal segments are re-subdivided into [(7/4)+ (7/28)] , i.e., [(1+ 1/2+ 1/4)+ (1/4)]= 2. We can notice that the subdividing process (see Figure 4b), actually, divides the 2nd segment that equals $1/7$ of the line-*S* (*AB*) into [(1/2+ 1/4)+ 1/4]. Hence, the answers that do not yield unit fractions for the second segment ($1/n$) are unsuitable for all cases in the $2/n$ table; see some of these incorrect answers (that do not agree with the 1st proviso-i) in the discussions of Gillings (1982, pp.52-69).

Number of mean fingers in the line-*S*	Minimum alternate S, [(n+1)/2]	Maximum and best alternate *S*	The measurable subdivisions in the mean-finger, and the corresponding values in the line-*S* (length-*S*)															The sum of two parts
n	*mi*	*mx*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
3	2	2	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	
5	3	3	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	
7	4	4	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	
9	5	6	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	
11	6	6	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	
13	7	8	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	
15	8	10	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	
17	9	12	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	
19	10	12	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	
21	11	14	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	
23	12	12	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	
25	13	15	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	
27	14	18	54	81	108	135	162	189	216	243	270	297	324	351	378	405	432	
29	15	24	58	87	116	145	174	203	232	261	290	319	348	377	406	435	464	
31	16	20	62	93	124	155	186	217	248	279	310	341	372	403	434	465	496	
33	17	22	66	99	132	165	198	231	264	297	330	363	396	429	462	495	528	
35	18	30	70	105	140	175	210	245	280	315	350	385	420	455	490	525	560	42
37	19	24	74	111	148	185	222	259	296	333	370	407	444	481	518	555	592	
39	20	26	78	117	156	195	234	273	312	351	390	429	468	507	546	585	624	
41	21	24	82	123	164	205	246	287	328	369	410	451	492	533	574	615	656	
43	22	42	86	129	172	215	258	301	344	387	430	473	516	559	602	645	688	
45	23	30	90	135	180	225	270	315	360	405	450	495	540	585	630	675	720	
47	24	30	94	141	188	235	282	329	376	423	470	517	564	611	658	705	752	
49	25	28	98	147	196	245	294	343	392	441	490	539	588	637	686	735	784	
51	26	34	102	153	204	255	306	357	408	459	510	561	612	663	714	765	816	
53	27	30	106	159	212	265	318	371	424	477	530	583	636	689	742	795	848	
55	28	30	110	165	220	275	330	385	440	495	550	605	660	715	770	825	880	
57	29	38	114	171	228	285	342	399	456	513	570	627	684	741	798	855	912	
59	30	36	118	177	236	295	354	413	472	531	590	649	708	767	826	885	944	
61	31	40	122	183	244	305	366	427	488	549	610	671	732	793	854	915	976	
63	32	42	126	189	252	315	378	441	504	567	630	693	756	819	882	945	1008	
65	33	39	130	195	260	325	390	455	520	585	650	715	780	845	910	975	1040	
67	34	40	134	201	268	335	402	469	536	603	670	737	804	871	938	1005	1072	
69	35	46	138	207	276	345	414	483	552	621	690	759	828	897	966	1035	1104	
71	36	40	142	213	284	355	426	497	568	639	710	781	852	923	994	1065	1136	
73	37	60	146	219	292	365	438	511	584	657	730	803	876	949	1022	1095	1168	
75	38	50	150	225	300	375	450	525	600	675	750	825	900	975	1050	1125	1200	
77	39	44	154	231	308	385	462	539	616	693	770	847	924	1001	1078	1155	1232	
79	40	60	158	237	316	395	474	553	632	711	790	869	948	1027	1106	1185	1264	
81	41	54	162	243	324	405	486	567	648	729	810	891	972	1053	1134	1215	1296	
83	42	60	166	249	332	415	498	581	664	747	830	913	996	1079	1162	1245	1328	
85	43	51	170	255	340	425	510	595	680	765	850	935	1020	1105	1190	1275	1360	
87	44	58	174	261	348	435	522	609	696	783	870	957	1044	1131	1218	1305	1392	
89	45	60	178	267	356	445	534	623	712	801	890	979	1068	1157	1246	1335	1424	
91	46	70	182	273	364	455	546	637	728	819	910	1001	1092	1183	1274	1365	1456	130
93	47	62	186	279	372	465	558	651	744	837	930	1023	1116	1209	1302	1395	1488	
95	48	60	190	285	380	475	570	665	760	855	950	1045	1140	1235	1330	1425	1520	
97	49	56	194	291	388	485	582	679	776	873	970	1067	1164	1261	1358	1455	1552	
99	50	66	198	297	396	495	594	693	792	891	990	1089	1188	1287	1386	1485	1584	
101	51	101	202	303	404	505	606	707	808	909	1010	1111	1212	1313	1414	1515	1616	

Figure 5. Explaining the architectonic method of the 2/n table in RMP, in 19 columns. c-1 shows the n mean-fingers in the line-S, i.e., the denominator under 2; c-2 shows the values of the minimum alternate S or m_i that equals [(n+1)/2], i.e., the minimum denominator for the 1st partition. c-3 shows the maximum and best alternate S or m_x, i.e., the denominator of the 1st partition. c-4 to c-18 show the 15 measurable values of subdividing the n mean-fingers in the line-S; denominators of the other partitions in the answer are colored with dark orange and the values that could be added together in one partition are colored with purple. c-19 shows values of the denominators of the 2nd partition, where each equal to (and a substitute for) the sum of the two partitions that their denominators are colored with purple. The cells in green color in the 1st row show the other possibility of subdividing 2/3 as was noticed by, e.g., Gillings (1982, p.53).

The answers for the rest of the odd numbers of equal parts *n* in the line-*S* (length *S*) in RMP's architectonic table of 2/*n* were found in a similar way, and in some answers (in case of 3 sub-segments or partitions) the Harpedonaptae added the last two partitions together, in one, in the final answer like the cases of 2/35 and 2/91 (see, Figure 5). This is only in case if the denominator of the substitute unit-fraction is not more than $2n$, and this is the 5th proviso-v. Regarding the line-segment 2/35, [(1/70)+ (1/105)]= 1/42; where 42 is less than 70; besides, 7 divides both 35 and 42, i.e., 42 is in the row of n=7 mean-fingers. Accordingly, in this case, 42 is the number of mean-fingers in a 3rd auxiliary line for re-subdividing the line segment 2/35. Alternatively, for shortening the subdividing time either in drawings or in the field, since 1/42= [(1/35) * (5/6)], one can imagine that the 1st auxiliary line of 35 mean-fingers is being re-subdivided into 42 sub-segments, where each is 5/6 mean-finger. Similarly, regarding the line-segment 2/91, [(1/182)+ (1/455)]= 1/130; where 130 is less than 182; besides, 13 divides both 91 and 130, i.e., 130 is in the row of n=13 mean-fingers. The only other (2nd and 3rd) partitions, in case of 3 sub-segments, that could be added together in the 2/n table are in the case of the line-segment 2/95, where [(1/380)+ (1/570)]= 1/228; and 19 divides both 95 and 228, i.e., 228 is in the row of n=19 mean-fingers. Gillings (1982, p.68) noticed the possibility of using 1/228 for decomposing 2/95; but, in this third case, 228 is more than $2n$ (i.e., 2*95), which does not agree with the 5th proviso-v.

4. THE CALCULATIONS OF SINE AND ANGLE VALUE IN RMP

Because the early philologists had dealt with nearly all the geometric symbols in RMP as alphabetical signs (Aboulfotouh, 2019), math scholars dealt with all the problems in RMP that do not include geometric figures as problems outside the realm of geometry. For example, they dealt with the eight problems: RMP#24-27 and RMP#31-34, in abstract way without involving any geometry, e.g., Clagett (1999, p.117& pp.141-143) concluded that RMP#24-27 are on finding an unknown quantity when an expression involving the unknown and fractions of it is specified. In the first sentence in these problems the philologists transliterated the geometric symbol of a circular horizon, i.e., , as a mast sign ; and they transliterated the geometric symbols of a right-angle triangle in quarter of a circle , i.e., , and half diameter, as signs of a mountain and a hand , respectively. Besides, the letter R under the symbol of the referenced module , i.e., modular (Aboulfotouh, 2012), and followed by a line (not the letter N) implies the length R as part of the module, and in case of dots implies as parts of the module.

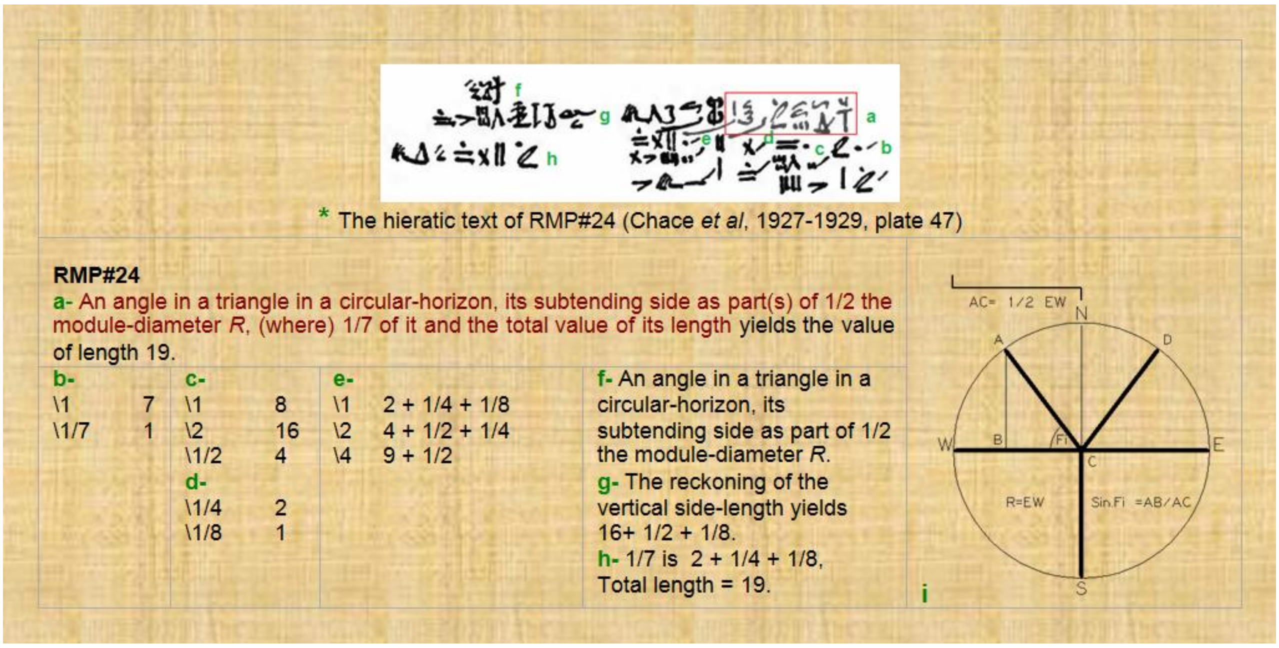

Figure 6. Translation of the hieratic text of RMP#24. The letters in green color, from "a" to "h", correspond to the same sections' letters beside the hieratic text. In "a", the text in red color is translation of the first sentence in the hieratic text that appears in a red box; and "i" shows the geometric figure that corresponds to reckoning the length of AB from 1/sine the angle φ and the length of the radius CA.*

As shown in Figure 6i, the hieratic text of RMP#24 speaks about the correlation between two lengths: the radius CA of a circle that equals 19 units, and the height AB of the right-angle triangle ABC, which is the subtending side of the angle φ. In RMP#24, AB was used as a base-length (or value X=1), where CA= (8/7) AB. This means that CA is composed of 8 modules that each = 19/8 = 2+ 1/4+ 1/8 units of length, and the 7 modules of AB (value X) are 16+ 1/2+ 1/8 units of length. Accordingly, sine φ = AB/CA = 7/8= 1/2+ 1/4+ 1/8, and 1/sine φ= 1+ 1/7.

Regarding the values of angles in degrees, RMP includes some examples, e.g., RMP#74 is on the double ratio between the lengths of two arcs. Because philologists thought that the angle symbol is the letter *R*, math scholars concluded that RMP#74 is also about loaves, see its hieratic text and translation in (Clagett, 1999, plate-96 & p.177). RMP#74's text says, in a right-angle triangle that its hypotenuse is a diameter of a circle (it is not the letter *K*), like the triangle *EAW* in Figure 6i, if the angle *AEW* is 5° and the length of its arc *AW* is 1000 units, these 1000 units are also the arc length of the angle *ACW* that equals 10°. Besides, if the angle *AEW* has been doubled, and became 10°, the length of its arc will be 2000 units and the related angle *ACW* will be 20° for the same arc length of 2000 units.

Hence, in RMP, the value of an angle in a circular horizon could be implemented in the field with the lengths of the radius (hypotenuse) and the subtending side of that angle, in addition to the spread-out length of its arc in measurement units, where each degree of arc represents a module of length.

5. SUBDIVIDING ARCS AND SIDES OF AN OCTAGON IN A GRID-SYSTEM IN RMP

The translation of RMP#65 is another example on dealing with the geometric symbols as alphabetical signs. Therefore, e.g., Clagett (1999, p.171) concluded that RMP#65 is on dividing 100 loaves among 10 men.

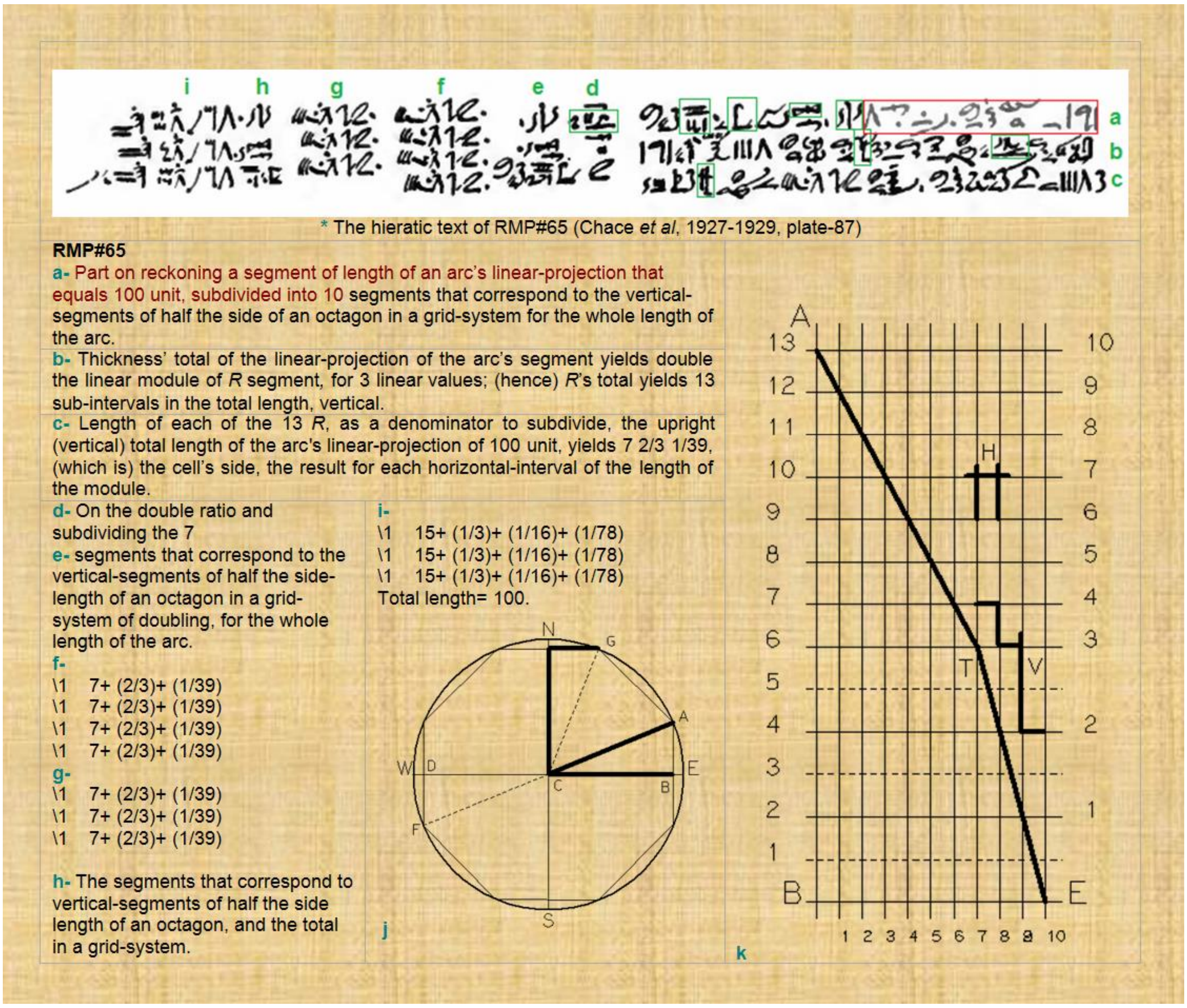

Figure 7. Translation of the hieratic text of RMP#65. The letters in green color, from "a" to "i", correspond to the same sections' letters beside the hieratic text, and both the mathematical and geometric symbols in the text appear in green boxes. The text in red color in "a" is translation of the first sentence in the hieratic text* that appears in a red box. j- shows the geometric figure of quarter of an octagon. k- shows a grid-system of 130 rectangles for plotting the arc ATE of half the octagon's side AB (in j); it also shows the symbols of: the horizontal intervals (marked H), and the diverse vertical intervals by doubling the length (marked V).*

This is because in the first sentence of RMP#65 the philologists thought that the geometric symbol of quarter of an octagon (*ABCNG* in Figure 7j) is the hieratic form of seated priest in hieroglyphs. In addition, they did not decode the geometric symbol of the two opposite triangles that each equals the area of 1/16 of an octagon (the triangles *ACE* and *FCW* in Figure 7j); where the arc of half the octagon's side was plotted in a grid-system of rectangles. They did not decode too other geometric symbols in the text that do mean: the diverse vertical intervals and the uniform horizontal intervals of the grid-system, correlating a segment to a vertical segment, and a segment of the arc's vertical projection. In short, RMP#65 is on subdividing the arc of half the octagon's side, where the arc's vertical projection (height) equals 100 units. The arc is subdivided into 10 segments that correspond to the lengths of 10 linear segments of half the octagon's side that equals the arc's height; where each of its three lower segments (of the height) is twice each of its upper seven segments.

Hence, in Figure 7k, the vertical line *AB* is subdivided into 13 equal intervals that each is one module, and the line *EB* is subdivided into ten equal intervals. The intervals of the arc's upper part from *T* to *A* correspond to 7 vertical intervals that each one is a vertical module, i.e., each is 7+ 2/3+ 1/39 units of length. The intervals of the arc's lower part, from *E* to *T*, correspond to 3 large vertical intervals that each one equals two vertical modules, i.e., each is 15+ 1/3+ 1/16+ 1/78 units of length. Moreover, based on what have been shown herein above on RMP#24, in Figure 7j, if the lengths of *CA*, *AB*, and *CB* are known, the values of the vertical intervals of *AB* and the horizontal intervals of *EB* could be reckoned in units; and accordingly, the surveyor can plot the arc (semi circular-curve) *ATE* in the field, using a similar grid-system. Besides, a mathematician can also reckon the perimeter of the circle (as diagonals of rectangles in the grid-system) and double-check too its reckoned area (in RMP#50) of 64 square units for the diameter of 9 units. Regarding the method in RMP#65, any architect can notice that the more measurable and diverse intervals he can use, the more accurate the length and the shape of the arc he can draw and implement in the field.

6. DISCUSSION

Based on what have been discussed so far, regarding the architectonic method of the 2/n table, the calculations related to the sine of an angle in RMP#24, and the idea of the grid-system in RMP#65, herein below, shows how it was possible for the pyramids designers to reckon and implement the tilt *a* of the Great Pyramid's entrance passage, using the arithmetic of unit fractions and the divisions of the ancient Egyptian cubit rod. According to Petrie's survey (Petrie, 1883, p.58), the tilt *a* of the entrance passage of the Great Pyramid in Giza Plateau was found equal 26° 31' 23" ±5". Besides, this author (Aboulfotouh, 2007 & 2015) showed that the pyramids designers of the 4th Dynasty (led by king *Suphis-I* and his grandson king *Ratoises* or *Idris* (Manetho, 1940-1964, pp.45-50)) assumed that Earth's obliquity range is from O_i= ~21° 40' 23" to O_x= ~24° 18', with a mean O_m= ~22° 59' 10". They encoded these three data in the design of the layout of the horizon of Giza Pyramids, together with the obliquity of time O_t= ~24° 6' (Aboulfotouh, 2014). Using the equation of M. Bessel (Nallino, 1911, p.270) & (The Penny Cyclopedia, 1840, p.495), Earth's axial tilt of ~24° 6' meets the year ~3055BC (Aboulfotouh, 2002, 2014, & 2015). In modern astronomy, the assumptions regarding Earth's obliquity range are, e.g., from 22.1° to 24.5° (Milankovitch, 1941), from 22.61° to 24.23° (Laskar, 1986, p.86), or from 22.5° to 24.5° (Meeus, 1991, p.135); besides, the current Earth's axial tilt is ~ 23.44° (in the descending phase).

Moreover, it was found that the assumed minimum obliquity O_i (~21° 40' 23") is the tilt of the lower entrance passage of the 2nd Giza Pyramid, and O_i/O_t = cosine the tilt angle of the upper entrance passage of that pyramid that equals ~ 25° 56' 4". According to the published survey data, these two values were found ~ 21° 40' and ~25° 55', respectively (Baedeker, 1908, p.129). Based on the Google Earth data, the current latitude λ of the Great Pyramid of Giza is 29° 58' 45". Using Eq.1 (Aboulfotouh, 2007) and the values of *a*, O_m, and O_i that are shown herein above, with and without the seconds of arc, as RMP#40 shows only the dividing of one degree of arc into 60 minutes (Aboulfotouh, 2019), the corresponding value of the used latitude λ of the Great Pyramid would be between 30° 00' & 30° 04'. This implies that the pyramid designer most likely used *a* = 30° ±1' 15", and sine *a* = ~ ½ instead, and as approximation, of ~ 599/1200 for *a* = 29° 58' 45". For reckoning the tilt *a* of the Great Pyramid's entrance passage, since,

Sin $a = [(\sin\lambda * \sin O_m)/(1-(O_i^2/O_t^2))^{1/2}]$

And knowing that,

O_t = ~ 24.10° = ~ (24+ 1/10)° = ~ (241/10)°

O_i = ~ 21.673° = ~ (21+ 1/2+ 1/6+ 1/156)° = ~ (3381/156)°

Sin O_m, i.e., *Sin* ~ 22.986° = *Sin* ~ (22+ 1/2+ 1/3+ 1/8+ 1/36)° = ~ 25/64

Sin λ, i.e., Sin ~ 30° ±1' 15" = ~ 30° ± (1/60+ 1/240)° = ~ 1/2

Then,

Sin a = [(1/2) * (25/64)]/[1- ((3381/156)/(241/10))^2]$^{1/2}$
Sin a = [25/128]/[1- ((5635)/(6266))^2]$^{1/2}$
Sin a = [25/128]/[1- (31753225/39262756)]$^{1/2}$
Sin a = [25/128]/[7509531/39262756]$^{1/2}$

Using the proposition of Gillings (1982, pp.214-217) on how to reckon the square root of a natural number from the table of multiplications, they could have found that the square root of 7509531 is between 2740 (square root of 7507600) and 2741 (square root of 7513081), where the former is close to 7509531. Similarly, the square root of 39262756 is 6266; then,
Sin a = [25/128]/[2740/6266]
Sin a = 15665/35072

To break up 15665/35072 into segments of unit-fractions, with applying the 1st proviso-i; since, 35072/15665= 2+ 1/5+ 1/26+ 1/2410, the minimum denominator m_i for the first partition (segment) is 3; but 3 will not yield a correct answer; then, 4 as alternate-*S* (or m_x) can decompose 15665/35072 into 9 partitions as follows:

\1	=35072
\¼	= 8768
\1/8	= 4384
\1/16	= 2192
\1/128	= 274
\1/1096	= 32
\1/4384	= 8
\1/8768	= 4
\1/17536	= 2
\1/35072	= 1
Total	= 15665

Because the denominators of the last 4 unit-fractions (segments) are not measurable (i.e., outside the range of n=101 mean-fingers in RMP's 2/n table), the denominator of total sum of the last 5 unit-fractions (47/35072) is between the two measurable numbers 744 and 747 in the auxiliary table (Figure 5). Since 8 divides both 35072 and 744, then,
Sin a = 15665/35072 = 1/4 + 1/8+ 1/16 + 1/128 + 1/744 (approximately)

The architect can then use the opposite of the procedures in RMP#24 and operate on a radius (hypotenuse) of, e.g., 10 royal cubits, in order to get the length of the subtending side of the tilt a from the hypotenuse of 10 royal cubits, as follows.

\1	= 10
\1/4	= 2+ 1/2
\1/8	= 1+ 1/4
\1/16	= 1/2+ 1/8
\1/128	= 1/16+ 1/64
\1/744	= 1/93+ 1/372

Total length of the subtending side = 4 + 1/4+ 1/8 +1/16 + 1/64 + 1/93 + 1/372 royal cubit.

Moreover, similar to what surveyors do today in the construction sites, one can draw on a vertical surface, quarter of a circle (like *NCE* in Figure 6i) that its radius equals 10 royal cubits (i.e., 2+ 1/2 royal fathom). Then draw a horizontal line at a height equals the subtending side of the tilt angle a, i.e., 4+ 1/4+ 1/8+ 1/16+ 1/64+ 1/93+ 1/372 royal cubit. To get the exact lengths of these fractions in the field from one royal cubit (28 mean-fingers), the surveyor can use two auxiliary lines: the length of the 1st line is preferably 8 or 16 mean-fingers for the binary fractions, and the length of the 2nd line is 31 mean-fingers for both 1/93 and 1/372. Then, the horizontal line will intersect with the arc at a point (like *D*), whereby the inclined line (like *CD*) from this point to the center of the circle represents the tilt a of the entrance passage of the Great Pyramid. The arc and the tilt line could be easily implemented in the field, in any scale, using a grid system like that in RMP#65, particularly if there are two tilts, as in the western entrance passage of the Bent Pyramid in Dahshure (Aboulfotouh, 2007).

Using decimal arithmetic, the tilt a of the Great Pyramid's entrance passage= ~ 26° 30' 59"; and the above fractional reckonings have yielded a = ~ 26° 31' 44", which is too close to the value in Petrie's survey: 26° 31' 23" ±5" (Petrie, 1883, p.58). Here, the difference ratio is ~ 1/2000, which is half the lower value of implementation tolerance (between 1/000 and 1/500) in modern steel structures. In this way, the ancient Egyptian Harpdonaptae (e.g., architects and surveyors) could have been able to reckon and implement the astronomical tilt of the entrance passage of the Great Pyramid in Giza plateau.

7. CONCLUSIONS

This paper showed that the mathematical information and the arithmetic of unit fractions in the so-called Rhind Mathematical Papyrus (RMP) that was copied, from the original source, for the Ancient Egyptian Harpedonaptae in circa 1550BC is consistent with and suitable for reckoning the tilt of the Great Pyramid's entrance passage in Giza plateau, from the retrieved complex equation. Regarding the translation of the related math text and the reckoning methods in RMP, the paper explained the following four findings for the first time. Firstly, it showed the meanings of some geometric symbols in RMP. Secondly, it showed the architectonic method of reckoning the 2/n table on subdividing the length of two equal line-segments from the measurable line-*S* (length *S*) of n equal segments, i.e., mean-fingers. Thirdly, it showed that RMP includes problems on angles and the sine of angles in right angle triangles, such as RMP#24 and

RMP#74. Fourthly, it showed that RMP#65 is on subdividing an arc that its vertical projection is half the side of an octagon in a grid-system of rectangles; the method that is suitable for plotting any circle, like the Giza Pyramids' horizon, in the field.

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