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ΒΩΜΟΣ ΔΩΔΕΚΑ ΘΕΩΝ / ALTAR OF THE TWELVE GODS: AN ASTRO-ARCHAEOLOGICAL ANALYSIS

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Astro-Archaeology Surveys

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ABSTRACT

The Altar of the Twelve Gods (ΒΩΜΟΣ ΔΩΔΕΚΑ ΘΕΩΝ) was constructed in the Athenian Agora, NW of the Acropolis, at the direction of Pisistratos the Younger, Archon of Athens, 522/521 BC (Thucydides). In 1934, archaeologists excavated a marble statue base *in situ* next to a rectangular limestone *hypaethral* edifice on the Panathenaic Way. Its inscription reads "Leagros, the son of Glaukon, made the dedication / To the Twelve Gods". Although the Altar is now buried under the Athens-Piraeus railway tracks, the author surveyed the exposed SW corner (12 September 2017) with a Gurley 20" theodolite (for solar observations) to Ground Truth published (magnetic compass) azimuths. This paper presents counter-evidence rejecting the interpretation that the Altar takes its orientation from the Panathenaic Way (Camp, 1986). Rather, the axis of the Altar's *peribolus* wall openings is oriented to horizon points consistent with Graeco-Roman *hypaethral* astro-architectural design principles (Vitruvius), viz.: Full Moon rise nearest the Midwinter Minor Standstill (+18.66° decl.) and set nearest the Equinox Minor Standstill (-5.1° decl.). The significance of these lunar standstill alignments is that they connote systematic *local* and *long-term* luni-solar observations in the century preceding the 19-year luni-solar cycle proposed by Meton of Athens (ca. 432 BC): i.e., the NE *peribolus* opening framed the last of a series of Midwinter Lunar Eclipses observable in Athens every 19-years, viz.: Partial Eclipse 29 December 560 BC; Total Eclipse 29 December 541 BC; and Partial Eclipse 30 December 522 BC— this last coinciding with the Altar's construction under Pisistratos the Younger.

KEYWORDS: Altar of the Twelve Gods, Athenian Calendar, Archaeoastronomy, eclipses, Greek architecture, Greek religion, Meton, Moon

1. INTRODUCTION

1.1 Background

Pisistratos the Younger, Archon of Athens 522/521 BC directed the Altar of the Twelve Gods (ΒΩΜΟΣ ΔΩΔΕΚΑ ΘΕΩΝ) be constructed in the Agora, NW of the Acropolis (Thucydides, 6.54). The Altar, also known as the Sanctuary of Pity, was the locus from which geographic distances from Athens were measured. In 1934, American archaeologists confirmed its location upon finding a marble statue base *in situ* inscribed "Leagros, the son of Glaukon, made the dedication /To the Twelve Gods" (Camp, 2010) (Figure 1).

Scholars have attributed the Altar's distinctive altar's architecture as the prototype for Imperial Rome's *Ara Pacis Augustae* (Müller, 1943; Thompson, 1952; Kleiner, 1992; Mauzy, 2006). Their shared architectural elements include: an altar enclosed by a rectangular wall (*peribolos*) with opposing twin openings; an axis of the openings running SW—NE; an enclosure "open to the sky" (*hypaethral, ὕπαιθρος*); and sited adjacent to a principal ceremonial avenue. The major difference between the two structures is that while only the limestone *peribolos* sill or foundation of Athen's Altar of the Twelve Gods remains, the *peribolus* of Rome's *Ara Pacis Augustae* retains its magnificent marble friezes. Recently, the "Tellus" frieze of the *Ara Pacis Augustae* has been reinterpreted as a sky map depicting the heliacal rise of the Morning Star phase of Venus (patron goddess of the Julian clan) seated with Aries and Taurus on the *peribolus* opening directly facing sunrise on the *Dies Natalis Romae* (21 April 9 BC), i.e. the 744th anniversary (or 93rd iteration of the 8-year Venus cycle) of the founding of Rome (753 BC) (Tiede, 2016). Therefore, if the Athenian altar were the architectural prototype for its subsequent Roman astro-architectural analogue, then it follows that the former may have been astronomically oriented as well; especially given the evidence to be found in Graeco-Roman literature, e.g.,

> *The early Greeks attached significance to the risings and settings of fixed stars and star-groups (Pleiades, Hyades, Orion, Sirius, Arcturus, etc.). Plato, Aristotle, Theophrastos all use risings of* stars *and solstices to specify times of the year….. Theophastos (Sign. I.4) tells us that Meton's teacher, Phaeinos, observed the solstices (τας τροπας) from Mount Lykabettos, and two other astronomoi from other mountains.* (Prichett, 2001).
>
> *…[O]ne who professes himself as an architect should be…acquainted with astronomy and the theory of the heavens…. From prescription (θεματισμώ), in the case of hypaethral (ὕπαιθρος) edifices open to the sky, in honor of Jupiter Lightning, the Heaven, the Sun,* ***or the Moon****: for these are the gods whose semblances and manifestations we behold before our eyes in the sky when it is cloudless and bright.* (Vitruvius, I:I,3; II:ii,5)

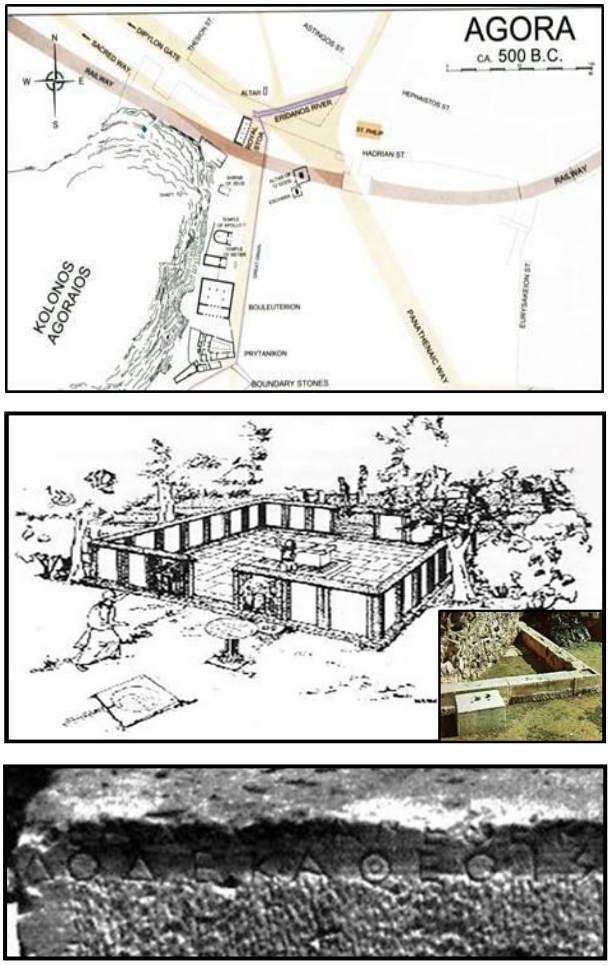

Figure 1. Agora plan; Altar XII gods reconstruction and SW corner photo; ΔΩΔΕΚΑΘΕΩΝ inscription. (Camp, 2010; McCallister, 1906; Meritt, 1966: http://www.greece-athens.com/page.php?page_id=258)

2. METHODOLOGY

2.1 Research Objectives

This paper investigates two research questions regarding the Altar of the Twelve Gods: (1) Does the Altar take "its orientation from the street" (Camp, 1986); and (2) Was the axis of the Altar's *peribolos* wall openings oriented to a coherent pattern of astro-calendric targets on the local horizon (Hawkins, 1968) when Pisistratos the Younger was Archon of Athens (522/521 BC)?

2.2 Research Design

Preliminary astro-archaeological data was compiled from the published 1935 site plans (Gadberry

1992) of the American School of Classical Study at Athens (ASCSA), NOAA Magnetic Field Calculator (IGRF12), NASA Index to Five Millennium Catalog of Lunar Eclipses, aerial/satellite imagery, and Google Earth's Digital Elevation Model (for horizon altitude). The (magnetic compass) azimuths plotted on published site plans are problematic due to the Altar's close proximity to the Athens-Piraeus railroad tracks potentially creating a local magnetic anomaly. Therefore the present study is based on the author's Ground Truth field survey with a Gurley 20" theodolite (for solar observations) and Garmin eTrex GPS, 12 September 2017 (Tiede 2017).

2.3 Ground Truth Survey Data

The field survey confirmed a 1° East local magnetic deviation resulting in a +4° declination increase of the NE horizon (south slope of Mt. Likavitos). Final astro-archaeological data are presented in Table 1 and the following: Latitude = North 37° 58′ 32.89″; Observer Elevation = 54 meters ASL; SW-NE Axis True Azimuth = 71.7°; Date = 522/521 BC; NE Altitude ≈ +5° 16.8′; and SW Altitude ≈ +13° 34.2′. Program *Stonehenge* (Hawkins, 1983) calculated the two respective horizon declination vectors for the axis centered on the Altar's SW and NE *peribolus* openings. The declinations of three historical astro-targets (i.e.: Jupiter, Sun and Moon) prescribed for *hypaethral* edifices were compared with the Altar's SW and NE axis horizon intercept declinations for both years of the archonship of Pisistratos the Younger (522/521 BC), and modeled in Starry Night Pro Plus-6 digital planetarium.

Table 1. Theodolite (Solar Observation) Survey: West side of SW corner, Altar the Twelve Gods, Agora, Athens, Greece

Date: 12 September 2017						
Crew: VRT						
http://aa.usno.navy.mil/data/docs/AltAz.php						
https://www.accuweather.com/en/gr/athens/						
Time: UT+3h (CEE Time Zone B +2h +1h Summer Time)						
Local Time h m	**USNO AZ dd**	**Interpol. AZ dd**	**Obs. AZ dd**	**Δ AZ dd**	**Obs. AZ °**	**Obs. AZ '**
14:00	196.9					
14:00:07		196.9	215.30	-18.4	215	18
14:01	197.4					
14:25	207.1					
14:25:09		207.1	225.42	-18.3	225	25
14:26	207.5					
14:29	208.6					
14:29:50		209.0	227.17	-18.2	227	10
14:30	209.0					
14:34	210.5					
14:34:34		210.7	228.90	-18.2	228	54
14:35	210.9					
14:51	216.6					
14:51:21		216.8	235.03	-18.3	235	2
14:52	216.9					
14:58	219.0					
14:58:47		219.2	237.50	-18.3	237	30
14:59	219.3					
15:04	220.9					
15:04:36		221.1	239.40	-18.3	239	24
15:05	221.3					
			Mean Average	-18.3	-18	-17
			+	360.0		
		SE-NW Axis True Azimuth		**341.7**	341	43
			-	270.0		
		SW-NE Axis True Azimuth		**71.7**	71	43
	Temperature (°C)			Long	Lat	
Date	High	Mean	Low	East	North	
12-Sep-17	32	27	22	23° 43'	37° 58'	
Historical	28	24	19			

3. ANALYSIS

3.1 Street Orientation Hypothesis

If, as Camp (1986) states, the Altar of the Twelve Gods "…takes its orientation from the street ", then the NE and SW sides of the Altar should be parallel to the extant curbing of the Panathenaic Way immediately adjacent. However, Figure 2 and three other published archaeological site plans of the Agora show that the Altar is not parallel to the Panathenaic Way, but is instead rotated approximately 31° from the adjacent curbing (see Crosby, 1949; Gadbery, 1992; Mauzy, 2006). Such a large deviation is sufficient to reject the hypothesis that the altar takes its orientation from the street.

3.2 Astro-Orientation Hypothesis

Table 2 compares the NE and SW vectors of the *hypaethral* Altar's *peribolus* openings at the time of its construction (522/511 BC) with the respective contemporary declinations of the Sun and Moon disc centers (i.e., disk radius = 0°). The horizon-target declinations differences for Jupiter and the Pleiades were out of range (Δ > 0.5° decl.) and are not included in this paper.

The Goodness-of-Fit of the luni-solar targets is less than the angular diameter (Δ <0.5° decl.) of the moon or sun with the exception of the rising Full Moon, 12 December 534 BC (where Δ = -0.93° decl.).

The NE opening of the Altar's *peribolos* wall is oriented to the south slope of Mount Likavitos. Possible astronomical targets include: (a) the sunrises 50 days after the Vernal and 53 days prior to Autumnal Equinoxes; and/or (b) the Full Moon rise nearest the Midwinter Minor Lunar Standstill (+18.66° decl., see Hawkins 1983) every 235 lunar Synodic months or 19 solar Tropical years, 534—496 BC. (Figure 3, Tables 2 & 3).

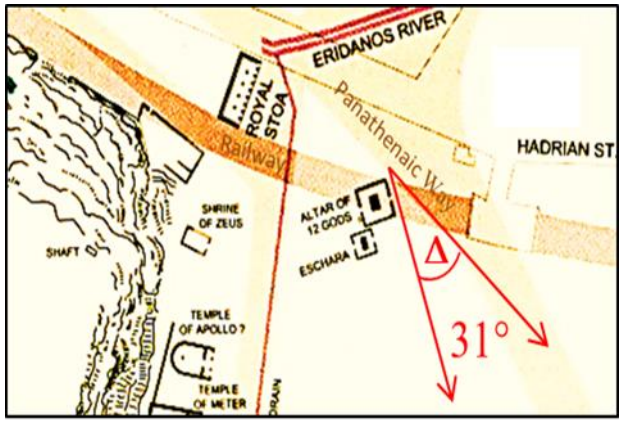

Figure 2. Orientation of the Altar of the Twelve Gods and Panathenaic Way, Athens (after Camp, 2010).

Table 2. Altar of the Twelve Gods Peribolus Wall Openings & Timing Luni-Solar Alignments

Target	Azimuth	Δ	Altitude	Δ	Declination	Δ	Local	Day-	Year	Year	
NE Horizon	°	°	°	°	°	°	Time	Month	BC	Δ	Interpretation
Rising	71.71		5.25		18.15						NE Peribolus Wall Opening Vector
Sun	71.71	0.00	5.64	0.39	17.86	-0.29	0649	18-May	522		Sunrise Spring/Summer Cross Quarter Day?
Sun	71.71	0.00	5.36	0.11	17.68	-0.47	0659	11-Aug	522		Sunrise Summer/Fall Cross Quarter Day?
Full Moon	71.71	0.00	13.33	8.08	22.49	4.34	1932	30-Dec	522		Midwinter **Partial Lunar Eclipse** (Saros 55)
Full Moon	71.71	0.00	4.62	-0.63	17.22	-0.93	1840	12-Dec	534		Midwinter Minor Lunar Standstill
Full Moon	71.71	0.00	5.24	-0.01	17.71	-0.44	1844	12-Dec	515	19	Midwinter Minor Lunar Standstill
Full Moon	71.71	0.00	5.35	0.10	17.68	-0.47	1836	12-Dec	496	19	Midwinter Minor Lunar Standstill
SW Horizon											
Setting	251.71		13.57		-4.88						SW Peribolus Wall Opening Vector
Full Moon	251.71	0.00	57.56	43.99	22.74	27.62	0347	10-Jan	522		Midwinter **Total Lunar Eclipse** (Saros 45)
Sun	251.71	0.00	14.37	0.80	-4.99	-0.11	1807	14-Mar	522		Vernal Equinox (27 Mar) - 14 days (inclusive)
Sun	251.71	0.00	14.78	1.21	-4.84	0.04	1743	11-Oct	522		Autumnal Equinox (29 Sep) +14 days (incl.)
Full Moon	251.71	0.00	14.75	1.18	-4.74	0.14	0625	18-Mar	539		Vernal Equinox Minor Lunar Standstill
Full Moon	251.71	0.00	14.97	1.40	-4.59	0.29	0624	18-Mar	520	19	Vernal Equinox Minor Lunar Standstill
Full Moon	251.71	0.00	15.24	1.67	-4.41	0.47	0624	18-Mar	501	19	Vernal Equinox Minor Lunar Standstill

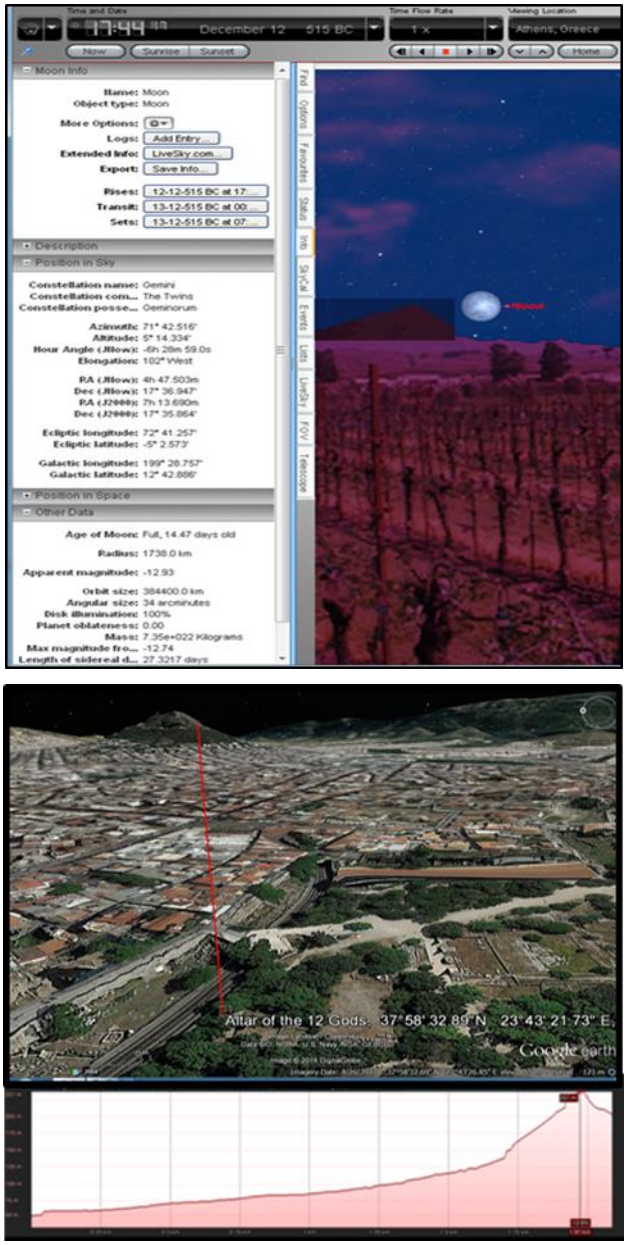

Figure 3. Full Moon nearest the Midwinter Minor Lunar Standstill rises from slope of Mount Likavitos, 12 December 515 BC. (Starry Night Pro Plus-6; Google Earth Pro)

The SW *peribolus* opening axis (True Azimuth = 251.7°) is oriented to the nearby Kolonos Agoraios ridge (Altitude = +13.57°) between the Shrine of Zeus and Temple of Apollo (ca. 500 BC) where the Full Moon nearest the Equinox set at Minor Lunar Standstill (-5.15° decl., see Hawkins 1983) every 235 lunar Synodic months or 19 solar Tropical years, 539—501 BC (Figures 2 & 4; Tables 2 & 3).

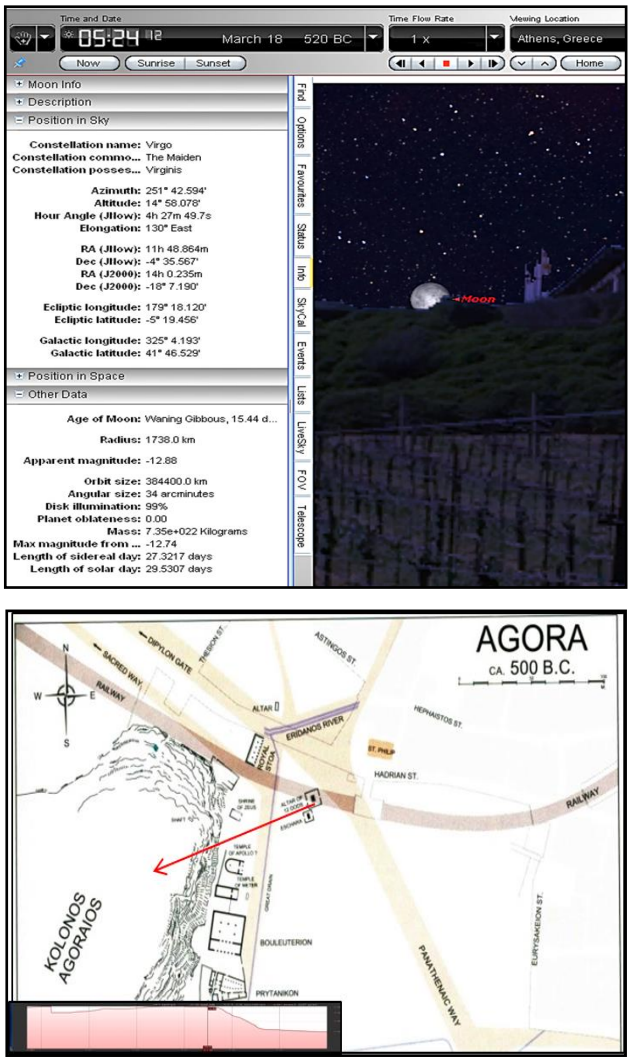

Figure 4. Full Moon nearest Equinox Minor Lunar Standstill sets over Kolonos Agoraios, 18 March 520 BC. (Starry Night Pro Plus-6; after Camp, 2010)

Table 3. Luni-Solar Periodicities (after Hawkins, 1973; Sarton, 1952; https://eclipse.gsfc.nasa.gov/SEsaros/SEsaros.html; https://eclipse.gsfc.nasa.gov/SEsaros/SEperiodicity.html#section104

	M	T	E			Halley's
	Lunar	Solar	Luni-Solar	Regression		Saros
	Synodic	Tropical	Eclipse	Lunar	Meton's	Eclipse
Unit	Month	Year	Year	Nodes	Cycle	Cycle
Days	**29.53**	365.24	346.62	6793.85	6939.69	**6585.78**
M	1	**12.37**	**11.74**	230.06	**235**	**223.02**
T	12.37	1	0.95	**18.60**	**19**	**18.03**
E	11.74	**0.95**	1	17.65	**18.03**	**19**

If the Altar's *peribolus* NE and SW openings were oriented to the Midwinter moonrise and Equinox moonset at Minor Lunar Standstills, then might the purpose of the Altar have been to honor the Moon goddess Selene (*Σελήνη*), or perhaps even predict the periodic cosmic combat between Zeus and Typhon, i.e., lunar eclipses (Plutarch, ca. AD 120?) Curiously enough, a Total Lunar Eclipse (Saros 45) passed over Athens as the Midwinter Full Moon, 10 January 522 BC, six months prior to Pisistratos' one-year archonship (Figure 5; Table 2). Twelve months later (ancient Greeks counted inclusively) a second Midwinter Full Moon emerged as a Partial Lunar Eclipse (Saros 55) from behind Mount Likavitos and passed over (Altitude = +13° 20′) the marble altar and NE opening of the *peribolus* wall at sunset on 30 December 522 BC, i.e., in the middle of his archonship (Figure 6; Table 2).

The *hypaethral* Altar of the Twelve Gods (***dodeca-theon***, ΔΩΔΕΚΑΘΕΩΝ) may have been named to honor Heaven (vs. the 12 Olympian gods); inspired perhaps by the remarkable twin midwinter lunar eclipses of 522 BC bracketing the ***12-month*** (counting inclusively) cycle of Heaven through all ***12 signs*** of the Zodiac (cf. Camp, 1986). In Greek, "*Zodion* could mean either a sign of the zodiac or a constellation of the zodiac; when the Greeks wanted to be perfectly clear they used the word ***dodeca****temorion* [**twelfth** part] for a sign of the zodiac" (Thurston, 1994).

4. CONCLUSION

Map inspection confirms that the *peribolus* of the Altar of the Twelve Gods is some 31° azimuth more than would be required to "take its orientation from the street" curb of the Panathenaic Way. Therefore, alternate rationales for the Altar's orientation deserve consideration, e.g., the highest peak on the local horizon (i.e., Mt. Likavitos summit, or Parthenon), or calendric/horizon astro-alignment. The evidence for topo-geomantic orientation is limited to the NE vector intersecting the south *slope* (as distinct from the *summit*) of Mt. Likavitos, while the SE axis intersects the top of the relatively flat Kolonos Agoraios ridge. By contrast, an astronomical rationale is supported by the Altar's twin reciprocal Minor Lunar Standstill alignments with horizon altitudes >+5° (to minimize the effects of atmospheric refraction), as well as by primary historical evidence. According to Graeco-Roman architectural design prescription, *hypaethral* edifices were constructed explicitly to honor Jupiter Lightning (*Ζεύς*), the Heaven (*οὐρανός*), the Sun (*Ἥλιος*) ***or the Moon*** (*Σελήνη*). Moreover, by the Vth century BC, "Scientific astronomy, we might say, was Greek or perhaps late Babylonian, Chaldean" (Sarton, 1952). For example, Anaxagoras of Clazomenae ("Lord of the Agora," ca. 510—428 BC, the pre-Socratic natural philosopher who moved to Athens from Ionia) suggested that "the eclipses of the moon are caused by the shadow of the earth and other bodies; and that eclipses of the sun are caused by the moon." Meton of Athens (ca. 432 BC) proposed that 235 synodic lunar months closely approximate 19 solar tropical years, and thereby correlated luni-solar calendric cycles (Table 3; Thurston, 1994).

Given its historical context, the construction of a VIth century BC Athenian *hypaethral* edifice with reciprocal NE and SW vectors aligned to the Full Moon at the Minor Lunar Standstills nearest Midwinter (+18. 66° decl.) and Equinoxes (-5.1° decl.) appears more likely the product of deliberate astro-architectural prescription, than of either coincidence (cf. Schaeffer 2007) or topo-geomancy. One might even imagine that Athenian observations of minor lunar standstills in the VIth century were used by Meton to supplement Babylonian data for deriving or confirming his theory of a 19-year luni-solar period (cf. Greswell 1862; Sarton, 1952; Thurston, 1994).

In conclusion, the Altar of the Twelve Gods deserves recognition as monumental architecture to honor the Cosmos (see Prichett, 1947, 2001; Prichett and Neugebauer, 1947); if not as astro-architecture to time Midwinter celestial clashes between Zeus and Typhon (Plutarch, ca. AD 120) observed as lunar eclipses by Athenians in the preceding century at 19-year intervals:

Partial Lunar Eclipse 29 December 560 BC (Saros 35);
Total Lunar Eclipse 29 December 541 BC (Saros 45);
Partial Lunar Eclipse 30 December 522 BC (Saros 55)

— this last coinciding with the Altar's construction under Pisistratos the Younger, Archon of Athens, 522/521 BC (Figure 6).

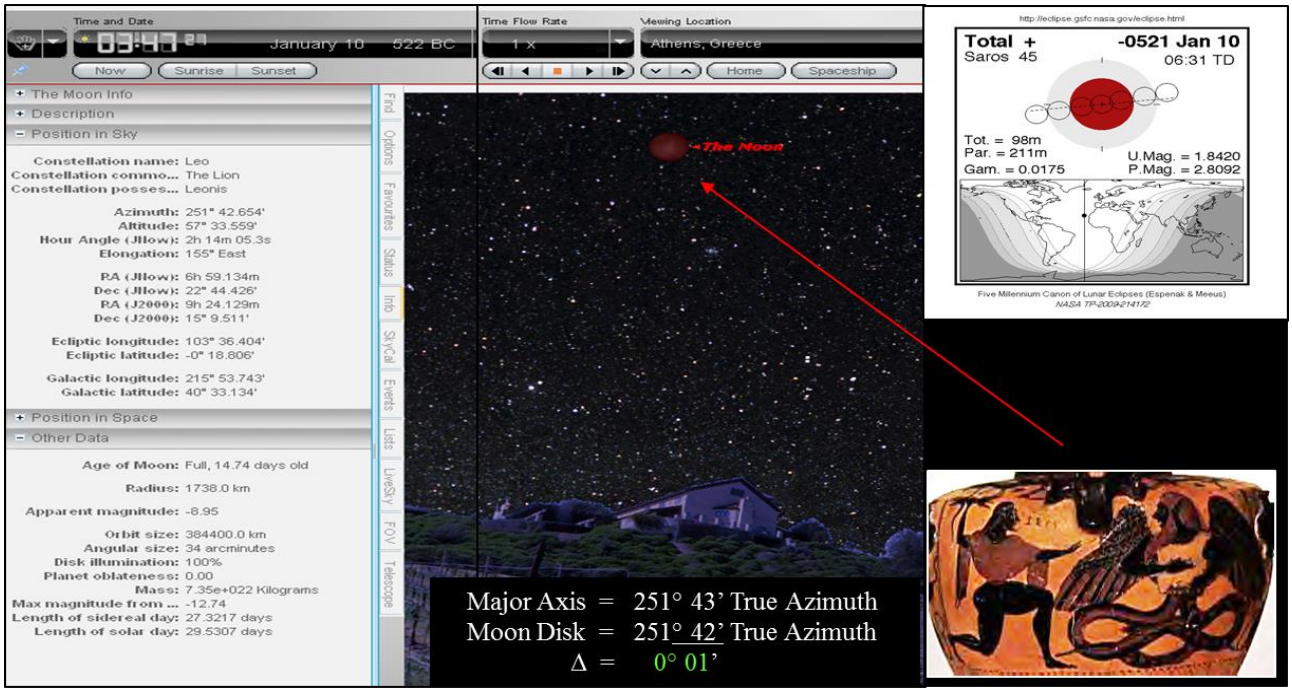

Figure 5. Typhon as Total Lunar Eclipse at Midwinter Full Moon over Athens, 10 January 522 BC. (Starry Night Pro Plus-6; https://eclipse.gsfc.nasa.gov/LEcat5/LE-0599--0500.html; Greek vase from Etruria, Italy, ca. 525 B.C., Staatliche Antikensammlungen Nr. 596, Munich)

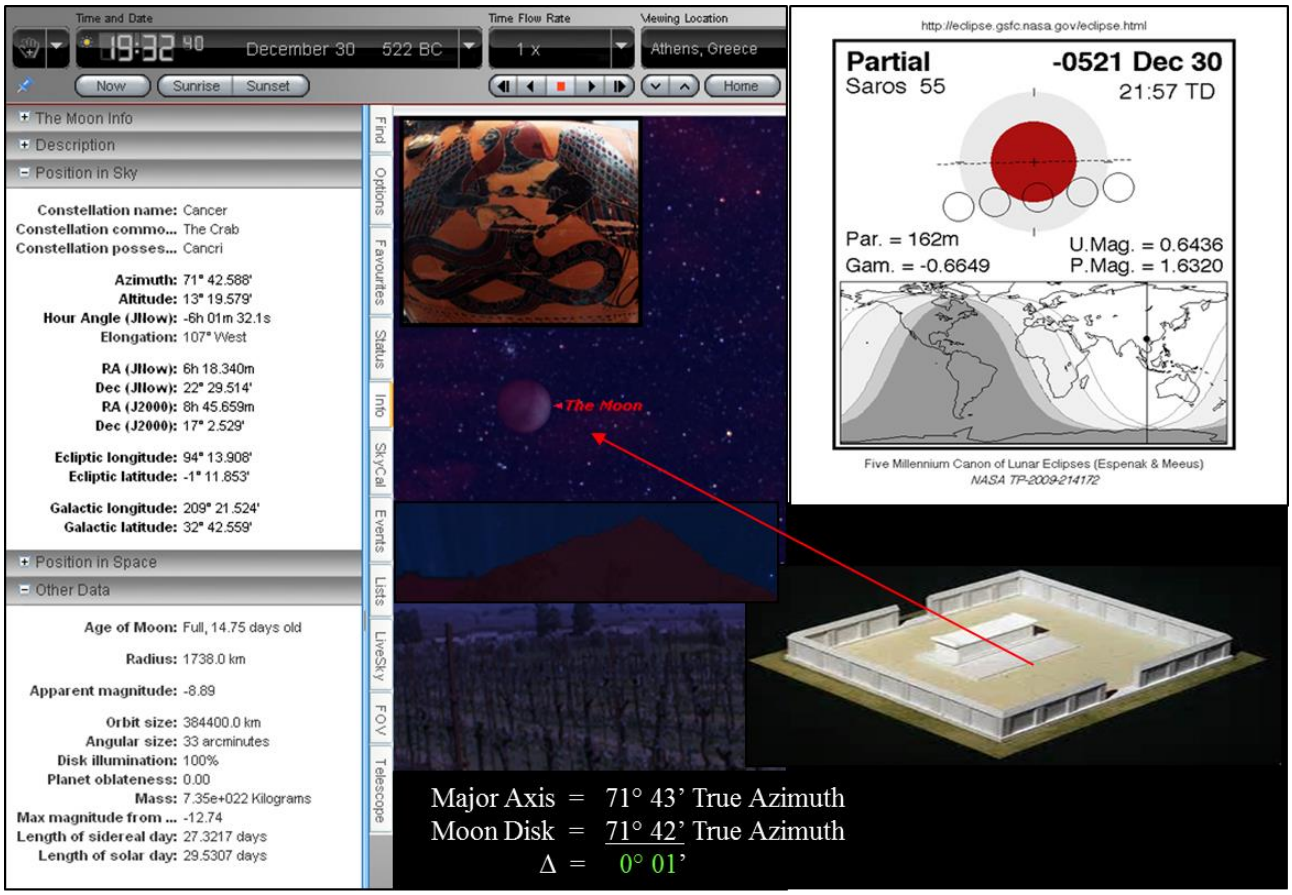

Figure 6. Typhon as Partial Lunar Eclipse at Midwinter Full Moon over NE peribolus opening, 30 December 522 BC. (Starry Night Pro Plus-6; https://eclipse.gsfc.nasa.gov/LEcat5/LE-0599--0500.html) Greek vase from Etruria, Italy, ca. 525 B.C., Staatliche Antikensammlungen Nr. 596, Munich)

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