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SOLAR GEOMETRY AND THE ORGANIZATION OF THE ANNUAL CYCLE THROUGH ARCHITECTURE AND THE FUNERARY LANDSCAPE IN QUBBET EL HAWA

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ABSTRACT

The longitudinal axes of the Middle Kingdom tombs excavated in the necropolis of Qubbet el-Hawa were precisely oriented to the summer and winter solstices. Located on the western side of the Nile, opposite Elephantine Island, the architectural design of these tombs differed greatly from that of the Old Kingdom with elongated spaces around these axes in relation to the solar cycle. As architecture, was excavated from the landscape itself, the presence of the sun was decisive in letting light in through the single doorway, which acted as a transitional threshold to project the sunlight indoors.
Funerary complex No.33 followed the longitudinal direction of the intersolstice axis. This important architectural construction of the necropolis reflects the evolution of a typical Upper Egyptian funerary model during the Middle Kingdom. Analysis of the illumination of the architectural space throughout the year confirms that the continuous movement of the sun during its cycle can be observed inside. The starting point of this analysis, is these spatial results, aiming to find an explanation for the geometric composition and specific design of the different architectural elements which make this a rounded, beautiful and harmonious complex. The QH33 funerary complex was built following very specific planning, which reworked and refined a model of a community which aimed to connect the celestial geometry to the geographical landscape through this architecture.

KEYWORDS: Egyptian architecture, archaeology, landscape, solar geometry, calendar, Middle Kingdom, necropolis, illumination, solstice, tombs, sunlight, elephantine, sanctuary

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1. INTRODUCTION

Elephantine, the capital city of the southernmost province of Upper Egypt until the Roman period, was located on the largest island north of the First Cataract. On the western shore, high ranking officials and members of the Elephantine elite chose the east-facing hill of Qubbet el-Hawa as a burial ground during the Sixth and Twelfth Dynasties (2305-1760 BC) (Hornung, 2006). QH33 is one of the most remarkable of the approximately, one hundred funerary complexes and rock-cut tombs which have been found, dates from the reign of Amenemhat III (1818-1773 BC) (Edel, 2008).It part of a set of tombs (QH30, QH31, QH32, QH33, QH34, QH34aa, QH34bb and QH34ee) (Fig. 01b) built between the reigns of Amenemhat II and Amenemhat IV (1878-1764 BC) (Jiménez & García 2017; Martínez et al, 2018). During this period, a succession of governors of the province, belonged to the same lineage that had begun during the reign of Senwosret I (1920-1875 BC) (Franke, 1994).

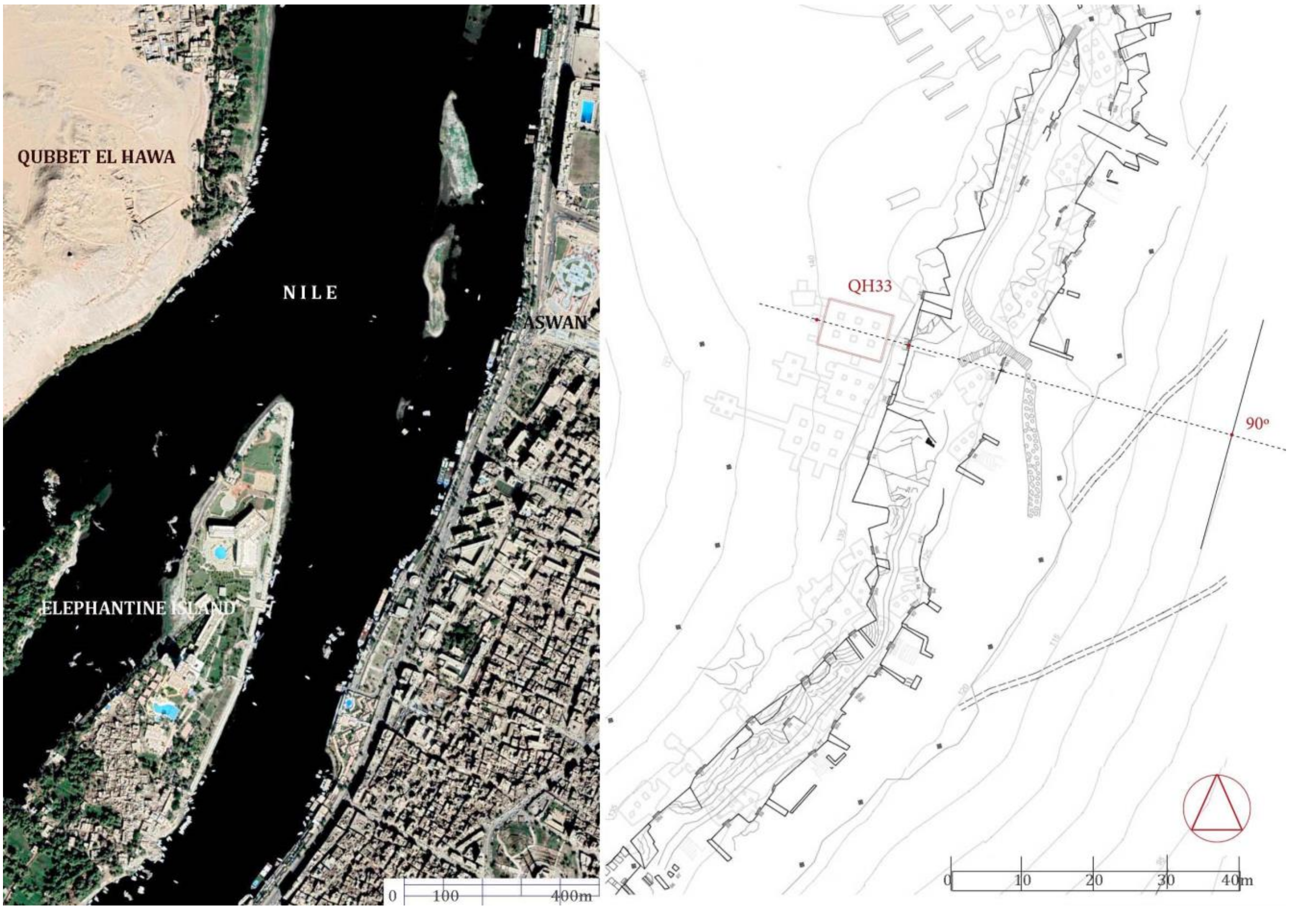

Figure 1(a) 1(b). Geographical location of the necropolis of Qubbet el-Hawa, Aswan (Egypt) (a) and location of funerary complex QH33 with longitudinal axis perpendicular to the Nile bed (b).

Since 2008, the University of Jaén has carried out a multidisciplinary excavation and study of these funerary monuments. Between 2008 and 2018, these focused on tomb QH33, discovered by Budge in 1885, although not excavated (Budge, 1887) and first described in 1960 (Edel, 2008). Despite careful excavation of the entire funerary complex, it has not been possible to determine the name of the governor who originally built, since the main burial chamber was looted in ancient times and the remains of the burial goods were destroyed. However, the analysis of the material culture of the funerary complex of QH33 (Jiménez et al., 2012; Jiménez, 2016; Martínez & De La Torre, 2018) has established that the construction was occupied from the end of the reign of Senwosret III (1837-1819) and during the first years of the reign of Amenemhat III. It has also been found that two governors were buried in QH33, probably in succession. Only the name of Heqaib III (Sánchez & Jiménez, 2015) has been confirmed, buried in honor of the builder of the funerary complex, who was probably his half-brother Heqaib-ankh.

The QH33 funerary complex basically reproduces the architectural structure of other Twelfth Dynasty (Jiménez & García, 2017) complexes incorporating in areas of worship, shafts and burial chambers. In QH33 the worship area is made up of: a forecourt with stepped structures at the north and south ends;

a large courtyard delimited by a large perimeter wall; a large door cut out of the monumental façade; a chapel with pillars and a niche for the statue of the governor, where the main rituals for the deceased would have been performed (Jiménez, 2015). The chapel, designed to house the statue of the deceased, was the center of the monument and at the far end of the longitudinal axis that began at the door. On the other walls of the chapel, other burial chambers were arranged to be occupied by persons close to the governor (Jiménez & Forstner, 2020). The governor´s body was deposited in a burial chamber preceded by an anteroom, located at the bottom of a shaft 11.5-meter deep, and sealed with rubble. Later, a second niche with a 5-meter deep shaft, was built with two burial chambers destined for Heqaib-ankh's stepbrother and successor, Governor Heqaib III, , and Lady Gaut-Anuket (Jiménez, 2015).

The QH33 complex, latitude 24º06′ N and longitude 32º 53′E, is built 135.00 meters above sea level (45.00 m above river level). The main axis of the funeral structure is oriented toward the Nile River, perpendicular to the maximum slope, with a clear view to the east, on the low slopes of the valley. The necropolis, at latitude 24º10'[1] , is oriented toward the east with a major solar incidence throughout the year[2], and coincides with one of the extreme points of the suns´s cycle, the summer solstice and the rising of the star Sothis. These events occurring together were essential to establishing the first day of the Civil Year, with the overflowing of the Nile. After a period of invisibility, the star was very close to the horizon just before sunrise[3]. This conincidence[4] was clearly visible from Elephantine and eventually this star was connected to one of the main deities of the ancient city, Satet[5]. In Elephantine, the sun, at its maximum declination during the summer solstice almost reached the vertical axis when it cast barely any shadow on the horizontal plane. During the winter, it displayed an almost homogenous slope of close to 43º, increasing progressively until day without a shadow (Isler, 2001; Kelley, 2011). This day heralded the Flood, when the sun vertically traced an almost infinite slope in relation to the gnomon, a phenomenon known as zenith pass (Isler, 2001). Although all burial sites built at Qubbet el-Hawa since the Old Kingdom between the middle of the Sixth Dynasty and the end of the Twelfth Dynasty were essentially oriented to the east, the orientation of QH33 replicated the example of the early temple of the goddess Satet on the island of Elephantine (Wells, 1985), aligned at dawn of the winter solstice, far south of the sunrise on the horizon. This orientation process began with the ceremony of the stretching of the cord (Arnold, 1991), used for all major constructions since the First Dynasty at least (Toby, 2000). This procedure highlighted the importance of orientation as an essential geometrical part of planning a construction, especially excavated ones. Thus, while all the burial sites in the necropolis were east-facing, the direction of the axis of QH33 was adjusted precisely in relation to the adjoining burial sites, QH32 and QH31.

2. OBJECTIVES

The Middle Kingdom was a time of splendor for Egyptian culture, as seen from the impact of high quality architecture as a form of expression. Constructions were perfected, while their orientation and architectural composition reflected an extensive knowledge of the solar cycle and derived geometry. The sanctuary niche of QH33 was constantly lit all year as the design focused on the longitudinal section, following the lines of the triangle of shadows. The carved space was permanently lit by the sun, emphasizing its sacred status. Once the axis of symmetry had been oriented and the orthogonal outer limits established, the construction was measured and delimited on the upper stratum or horizontal plane. Removing rock down from the top, all the progressively excavated horizontal layers could be seen, showing how the opening for a point of light played a key role in the creating space. Considering QH33 as the materialization of a multifactorial process (geographical, astronomical, religious and social) integrated into a landscape with a powerful symbolic load, the following objectives are proposed:

- To demonstrate that the design and construction of the QH33 complex is the result of the evolution of a well-thought-out model for geometrical translation of the solar cycle. This makes use of a selection of positions and orientations combined with a specific composition of architectural elements to provide a medium for the associated ritual.
- To use drawings and images based on precise topographic measurements for the geometric and constructive analysis of the funeral complex.
- To propose a specific research methodology for Architecture in Archeology contained in a landscape, linking horizontal-temporal strata with vertical planes, in order to decode it as a language closely linked to the functionality of the building.

3. METHODOLOGY

The building in question is the space out of the hill, with a specific geographical location and orientation linking it to the solar cycle. The presence of light in architecture, especially excavated architecture, was essential to the functionality of the building, and to its conception and construction processes. Thus, lighting is the key starting point to analyze the space of QH33, which has a single entrance of natural light whose inner distribution is controlled, as the discussion will show. Following the sunrise line of the winter solstice and the sunset line of the summer solstice both location and orientation serve as starting points, showing how QH33 considers celestial geometry and the geographical landscape, with an axis that is perfectly perpendicular to the bed of the Nile at that point (Fig.01).

A total station was used at different points of the QH33 set for geometric definition obtaining values of 115.24° and 28.26° for the alignment of the main longitudinal axis and of the façade plane respectively (**Table 1**). The 3D model (Fig. 02) of the exterior and interior assembly was created using photogrammetry techniques combined with Terrestrial Laser Scanning (TLS) to be used as reference with a global system (WGS84). An angle of approximately 87° was measured between the longitudinal axis and the plane of the façade, representing a subtle deviation between the orientation of the excavated space and the plane of the façade, which this text aims to explain. All the measurements included were obtained employing an external topographic network, with a precision above 50´´.The coordinates of this network were obtained using Global Navigation Satellite System (GNSS) technology, referenced and the WGS84 system. The different orientations were corrected for the true north deviation caused by the projection used, due to the convergence of the meridians. Table 1 indicates the main orientations of the QH31, QH32 and QH33 complexes, which share the same terrace for horizontal access. To check the interaction of the sun with QH33, the coordinates of the sun's position (azimuth and elevation) were calculated in relation to the year 1825 BC, as the approximate date of the beginning of the excavation work (Mozas et al., 2020), based on the oldest material culture (mainly ceramic) found in the QH33 complex. This bears in mind, the direction of the longitudinal axis of space (115.24°), the axis of the sunrise of the winter solstice (116°), and of the star Sothis (110°-115°). The ephemeris shown below (**Table 2**) was obtained from two sources with different algorithmic systems, the Horizons platform of the NASA Jet Propulsion Laboratory (NASA, 2020) and the Solar Position Algorithm (SPA) of the National Renewable Energy Laboratory (NREL) (NREL 2020).

Both systems precisely determine the azimuth and solar elevation for the year in question, although minor discrepancies were detected between both measurements. Thus, for each day of the solar year, the azimuth and elevation angles corresponding to the geographical position selected for the QH33 set were obtained at thirty-minute intervals. The graph (**Table 2**) shows the solar orientations for different elevations of sunrise and sunset on the days of the solstice and equinox. The different elevations correspond to 0°, -6°, civil twilight, and -0.83°, considering the atmospheric refraction in the apparent position of the sun. We also analyzed the spatial lighting results obtained with the software (DIALux evo 8.1), allowing us to trace the path of the sun inside QH33, throughout its complete cycle. It was verified that the sunrise during the winter solstice, the maximum solar declination, occurred on January 5, 1825 BC, considering parameters such as the diameter of the solar disk and atmospheric refraction (-0.83°). This practically coincided with the orientation of the longitudinal axis of the hypogeum, for the apparent sunrise of the winter solstice, with a minimum difference of 0.74°. Considering the exact orientation of the axis of QH33, the sun was elevated at -2.3° above the horizon during the winter solstice, within civil twilight and very close to the apparent sunrise. The small variation in the orientation of the line perpendicular to the façade plane (118° 26´) could refer to the search for the helical rising of the star Sothis, coinciding with the orientation of the Elephantine temple, consecrated to the goddess Satet (Wells, 1985).

Table 1. Angular orientations (azimuths) obtained in tombs QH31, QH32 and QH33.

	AZIMUTH FAÇADE	AZIMUTH MAIN AXIS	AZIMUTH DIFFERENCE
QH 31	33.48°	122.57°	89.09°
QH 32	33.48°	123.79°	90.31°
QH 33	28.26°	115.24°	86.99°

Table 2. Azimuths during sunrise and sunset on QH33 in 1825 BC (Source: NASA solar ephemeris, 2020).

AZIMUTH ORTHO AZIMUTH SUNSET							
		SUN ELEVATION					
	date	0°	-0.83°	-6°	0°	-0.83°	-6°
winter solstice WS	05 JAN	116.41	115.99	113.58	243.60	244.01	246.43
spring equinox EQX	06 APR	90.28	89.91	87.60	269.93	270.30	272.64
summer solstice SS	09 JUL	63.63	63.19	60.42	296.36	296.79	299.57
spring equinox EQX	08 OCT	90.05	89.63	87.34	269.72	270.09	272.42

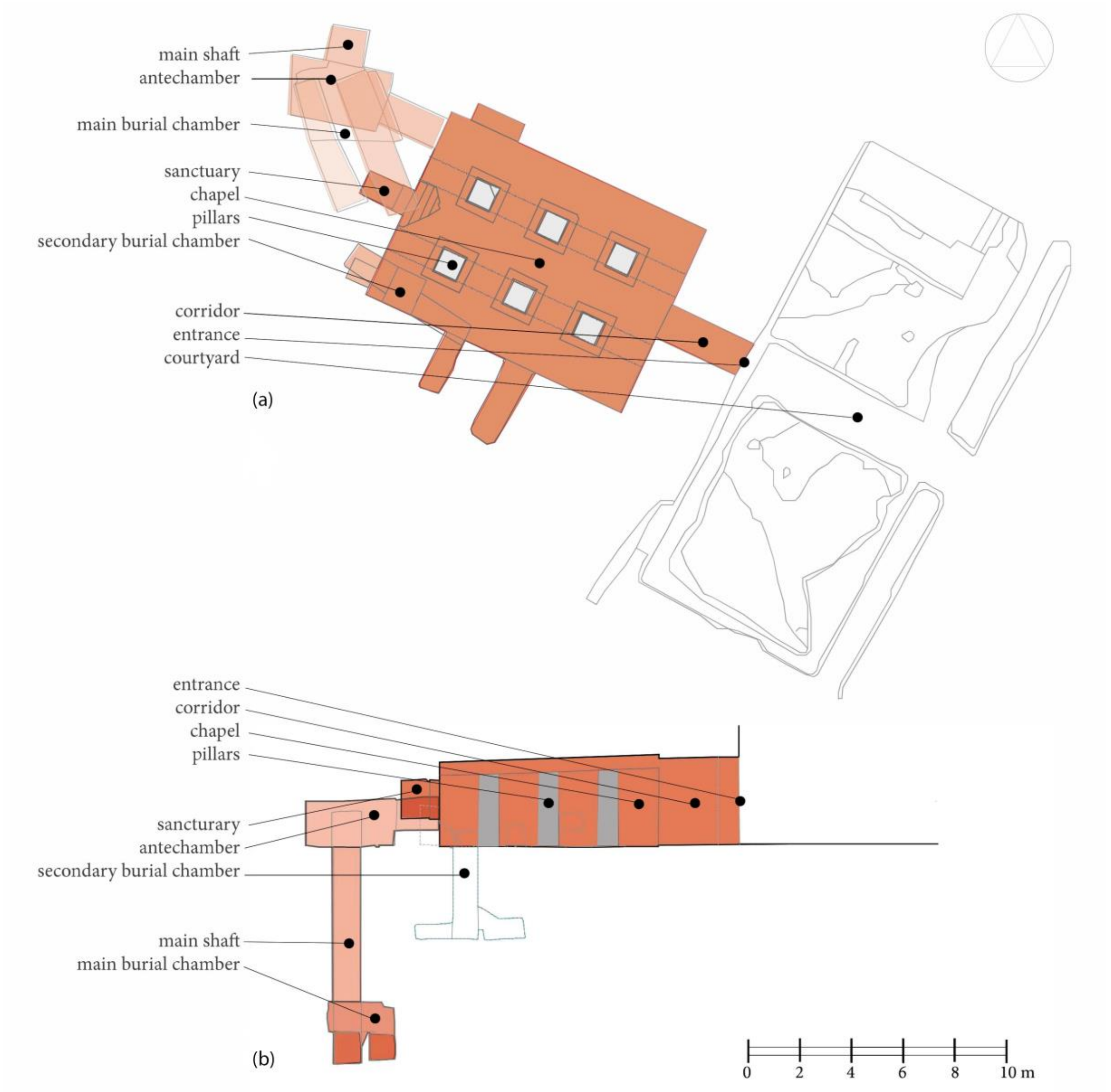

Figure 2. Plan (a) and longitudinal section (b) of the funerary complex QH33.

After establishing the orientations of the sunrise during the winter solstice, the solar ephemeris, and the geometry of the QH33 set a 3D model, plans, and sections were used to analyze the interaction of the sun and its path in the excavated space (Fig. 03). The winter solstice was one of the most important moments of the solar cycle in ancient Egypt, announcing the beginning of its rebirth and culminating in the summer solstice. The axis of the ensemble, coincides with the emergence of the sun, with a deviation of just 0.75º, directly illuminating the depository sanctuary of the deceased's Ka. The door cut out of the stone plane to project the light inside is wider than the niche (1.62m as opposed to 1.05m), compensating for the subtle deviation and ensuring that the sanctuary was fully illuminated at dawn on the winter solstice (Fig. 03a). It was not physically possible to determine the period of time during which the shrine or the statue were illuminated (Table 3) by recording the projection of the sun through the doorway for the different azimuth and elevation angles of the sun (Fig. 03b and 03c). This information was completed with the simulation results performed on the 3D model, as shown in Fig. 04. For about a month, around the winter solstice, the sanctuary was illuminated directly with the full intensity of the dawn sun. As the days grew longer, nearer the equinox, the sanctuary received the light dimly and remained in semi-darkness as the summer solstice approached, repeating the same cycle but in reverse. Thus, the space was designed as a dark chamber whose opening, the door, which successfully illuminated the central space throughout each sunrise with varying degrees of intensity. The three images shown reflect the progression in the change of season: from dawn, during sunrise and when the sun was rising further north (Fig. 04).

Different procedures were used to control the orientation of the longitudinal axis and to determine the main directions of the space. However, it can be assumed that recording the shadows projected by a vertical pole in the ground was a fairly intuitive proposal which must have been known long ago, prior to unification. The gnomon (Isler, 2001; Martin, 2015), from which the later sundials could be derived, was a simple instrument to record the length and direction of the shadows cast by the sun on the horizontal plane.

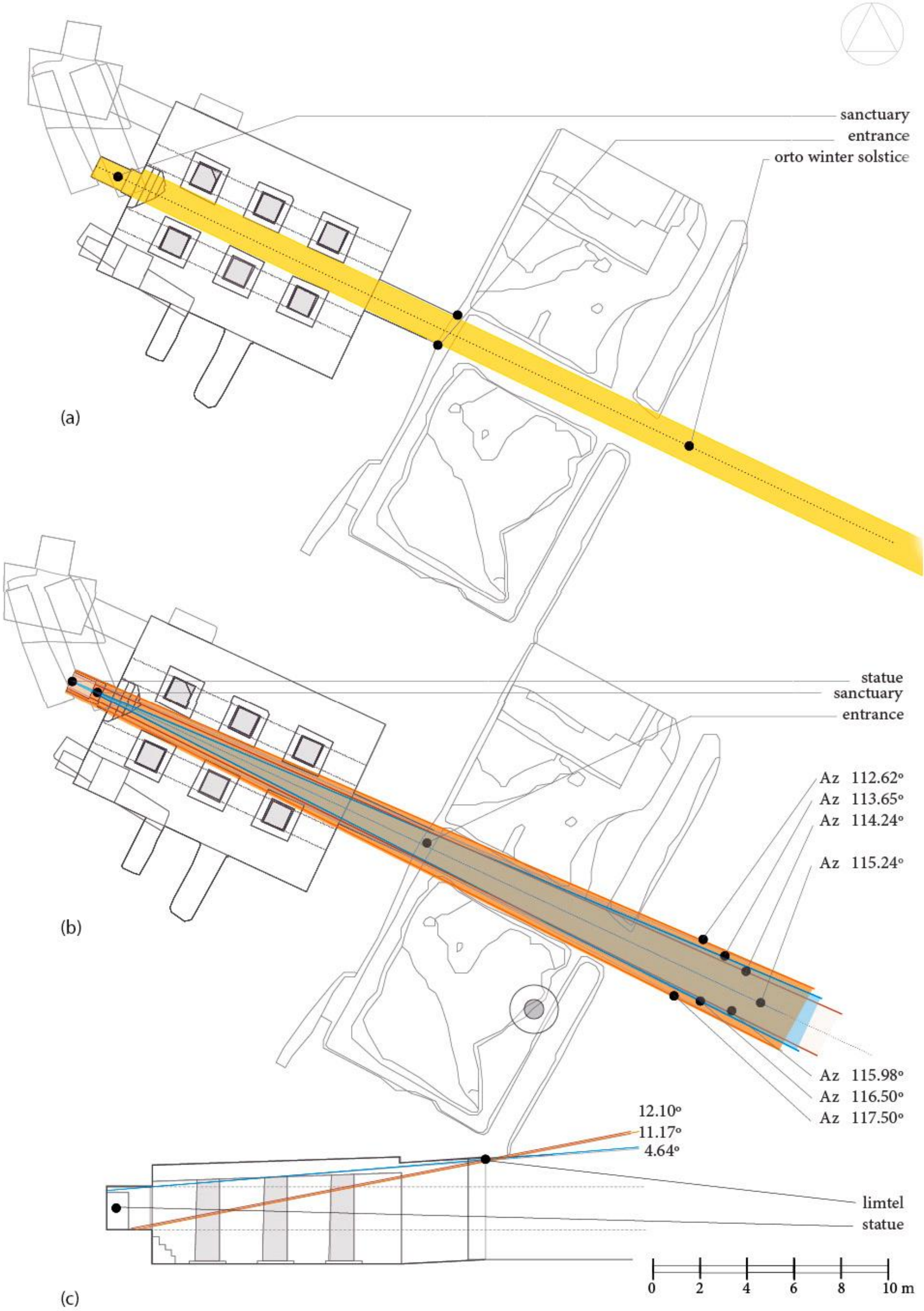

Figure 3. Plan (a), (b) and longitudinal section (c) of the funerary complex QH33.

Table 3. Azimuths during sunrise and sunset on QH33 in 1825 BC (Source: NASA solar ephemeris, 2020).

	BEGINNING	END	DAYS	% ANNUAL
whole sanctuary	15th DEC	26th JAN	41	11
statue	13th DEC	28th JAN	45	12
sanctuary central part	08th DEC	02th FEB	55	15

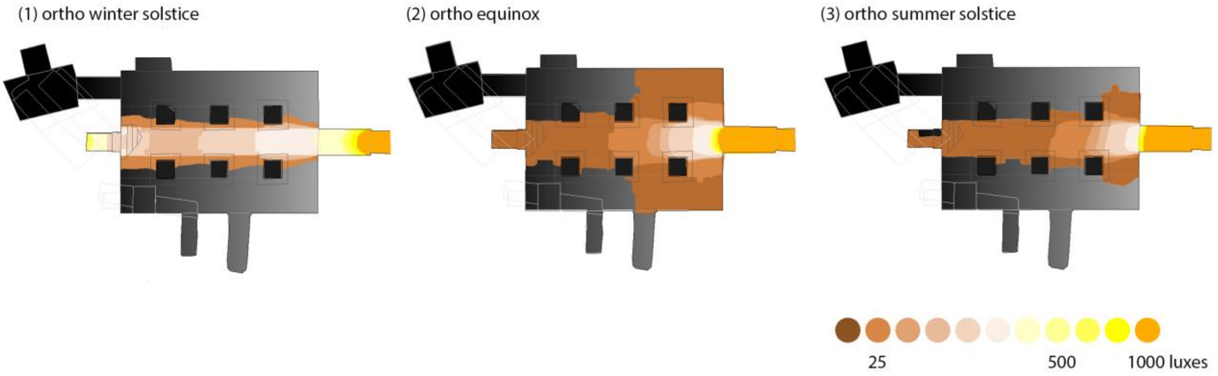

Figure 4. Scheme of illumination of the interior of the QH33 complex at (1) Ortho of winter solstice (WS), (2) Ortho of equinox and (3) Ortho of summer solstice (SS). Simulation with computer software (Dialux Evo). Light intensity scale in luxes.

Gnomon and shadow were linked by the sun, whose position was geometrically transferred relation to a specific place based on known trigonometric relationships (Aboulfotouh, 2022). When the shadow was equal to the height of the vertical pole, the balance between the vertical and the horizontal was reflected in the right-angle triangle drawn, which became an isosceles one (Fig. 05). As with the recording of the shadows, the movement of the sun inside QH33 could be analysed, using the access door to project the light inside, which explains its comprehensive orientation to the winter solstice. Transferring the solar movement to the interior of QH33 ensured a connection between the architecture of a sacred space and the rhythm of the cosmos using the vertical vector of the axis mundi (Mircea, 1959). Architecture expressed order and harmony through its geometry in such a way that it became a transitional space following the symbolic trajectory of the solar cycle (Westendorf, 1984). The recorded data were used to analyze the space of the QH33 set starting from the vertical section and the floor. This analysis of the axis of the tomb, included the reference to the star Sothis as herald of the Flood relation to the longest day and the zenith pass. Azimuth angles were calculated together with the elevation of the star for the year 1825 BC, and Stellarium[6] software was used to compare the position of the latter to that of the sun.

In this analysis of the orientation of the axis of the tomb, the reference to the star Sothis as herald of the Flood was included in the environment of the longest day and the zenith pass. Not surprisingly, the azimuth of the ascent of the star at dawn of the summer solstice was almost coincident with that of the winter solstice (Belmonte, 2003; Krauss, 1985). Azimuth angles were calculated along with the elevation of the star for the year 1825 BC, and Stellarium software was used to compare the position of the latter with that of the sun. Thus, for an arc of vision between the star and the sun greater than 8º it was confirmed that the heliacal ortho took place from July 9, coinciding with the solstice. The azimuth of Sirius varied between 110º and 115º depending on the elevation and precession of the equinoxes, estimated between 0º and 9º. This position made it possible to see the star from the niche where the statue of the deceased was located before dawn on the longest day of the year. This synchrony took place over several days (Figs. 3b and 3c)[7].

3.1. The vertical pole (gnomon) in relation to the celestial landscape and geographical location

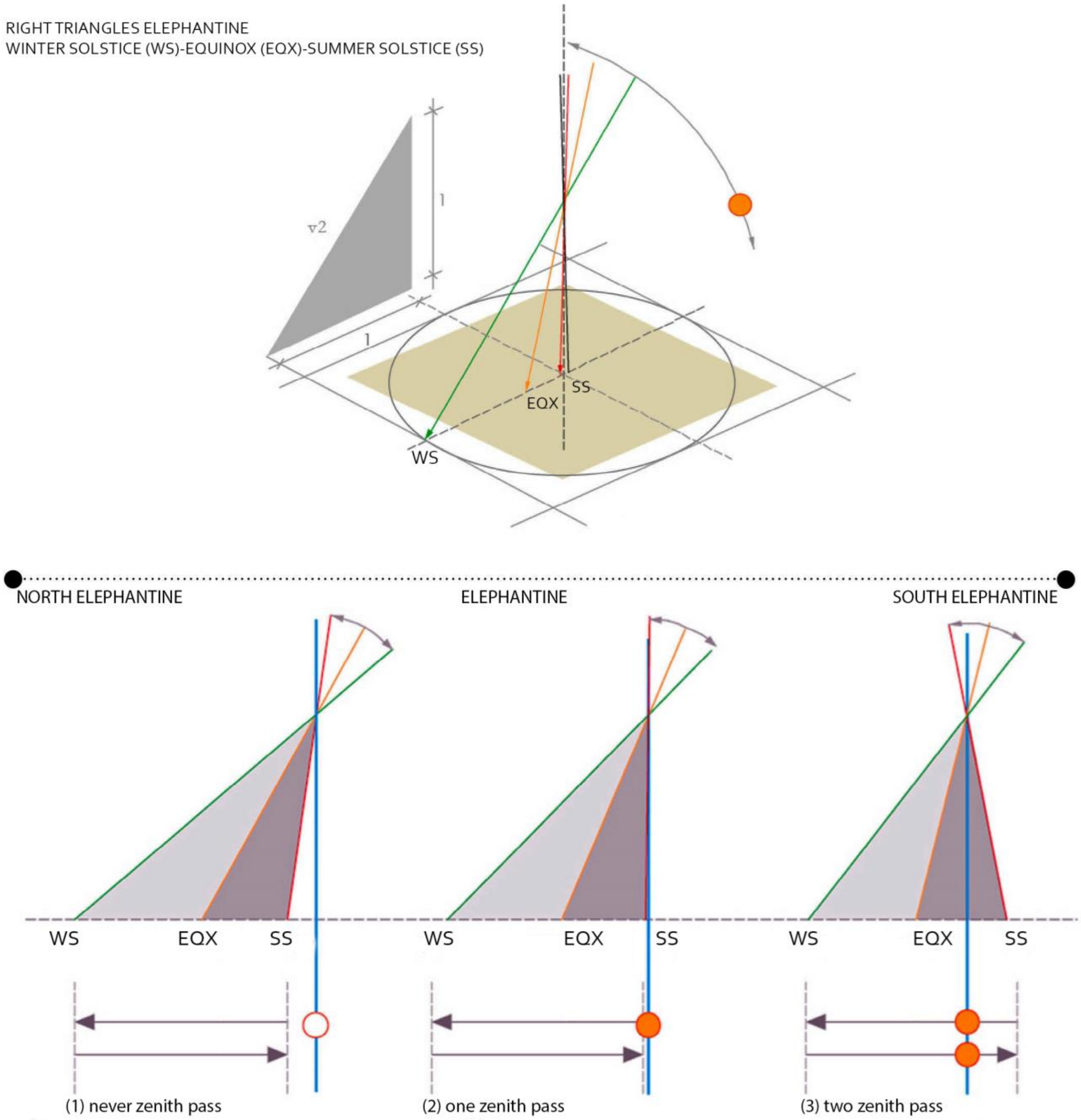

Figure 5. Shadow triangle on Elephantine throughout the solar cycle. (1) North of Elephantine, with no one zenith pass. (2) In Elephantine with an annual zenith pass. (3) South of Elephantine with two zenith pass.

In each geographical position, the gnomon represented the vertical axis connecting the center of the universe with the sun[8]. The geometry derived from the shadows required knowledge of the movement of the sun in a specific location. The vertical pole, approximately 2.00m high (about 4 royal cubits) cast its shadow, establishing a length and direction on the horizontal reference plane[9]. These records involved estimating the length of day and night (Lull, 2016; Magdolen, 2000; Parker, 1974), as well as the two directions of space and the four geographical orientations. The True North could also be determined in this way, together with the observation of the stars, which were always visible in the sky[10]. In order to record each sunrise on the horizon the two extreme points or foci of the solar path visible from specific latitude had to be established. Thus, each place established a unique relationship with the sun by drawing two characteristic right triangles for the solstices, the sun at its culmination point. The ideographic language of architecture, loaded with symbolism, was the perfect instrument to materialize these celestial events through geometry (Snodgrass, 2018). The shadow was a record of this relationship between the geographical landscape and the solar cycle, showing the inclination of the sun in relation to the surface. When the right triangle was an isosceles triangle, the moment was interpreted as a point of equilibrium between the vertical and horizontal proportions 1:1:√2 (Fig. 06). This relationship traced a slope or seked (Magdolen, 2000; Martin, 2015; Pérez, 2007) of (1/1), a reference for the measurement of the other shadows projected in the same place, successively growing fractally in harmony every noon[11]. It is quite possible that the slopes of the two most relevant moments, the solstices, were a geographical reference for the location of important constructions[12]. Thus, unique locations could be identified for certain buildings or specific geometric proportions could be established as a reference for the interrelation between the solar cycle and a specific geographical point (Pérez, 2007; Wright, 2009). Unsurprisingly, selecting the location and orienting the axis of the construction were essential decisions made during the founding ceremony (Arnold, 1991; Isler, 1989; Wright, 2009) and these involved the vertical and horizontal planes of the right triangle. As already seen, for the QH33 complex (Fig. 07) the winter solstice-sunset summer solstice-sunrise orientation traced its longitudinal axis to record the complete trajectory of the sun between its two extreme positions, constantly illuminating the excavated space every sunrise. The aim of this was to optimize the geographical position of the tomb in order to increase the amount and intensity of sunlight in space. In geometric terms, this orientation had to be considered as an equinox, and in turn, the plane of the ecliptic was corrected to capture the greatest amount of light.

From the winter solstice, the sun gradually began to rise rapidly toward the vertical axis, remaining longer in the vertical, increasing its slope, up to the summer solstice with a slope (1/0) close to infinity. The two extreme points of the sunrise divided the cycle into two stages, ascending and descending (Guénon, 1995; Snodgrass, 2018). Between winter solstice and the equinox, there were approximately six months of progressive daily ascent at the culmination of the sun, form when it began its descent until the shortest day of the summer solstice. During the first stage, from the winter solstice to the equinox the daily ascent to its zenith moved from approximately 44^o to 66^o, that is, from an almost unitary slope (1/1) to a slope of (1 / 0.25). However, for the next six months, between the equinox and the summer solstice, the slope grew rapidly until it approached the vertical (1/0). In term of geometry, the shadow cast by the gnomon was more elongated and diffuse in the first phase, and the direction and length changed more slowly than in the second phase. From the equinox to the summer solstice, it became more complicated to record the shadows, which shortened rapidly before reaching the longest day of the year, when there was hardly any shadow and the slope tended asymptotically to the vertical, towards infinity. The geometric and constructive rigor of QH33 complex had evolved in relation to the surrounding tombs (QH31 and QH32), and its orientation had been perfected, possibly as a consequence of the builders' exhaustive knowledge of the solar cycle and the owner's high social status within the community. It can be assumed that the small variation in orientation was intended to facilitate the vision of the winter solstice, together with the heliacal ascent of the pre-Flood star Sirius. Thus, although the arrival of the winter solstice was announced by the direct illumination of the niche and the Ka of the deceased, it was the sighting of the star, which indicated the arrival of the summer solstice.

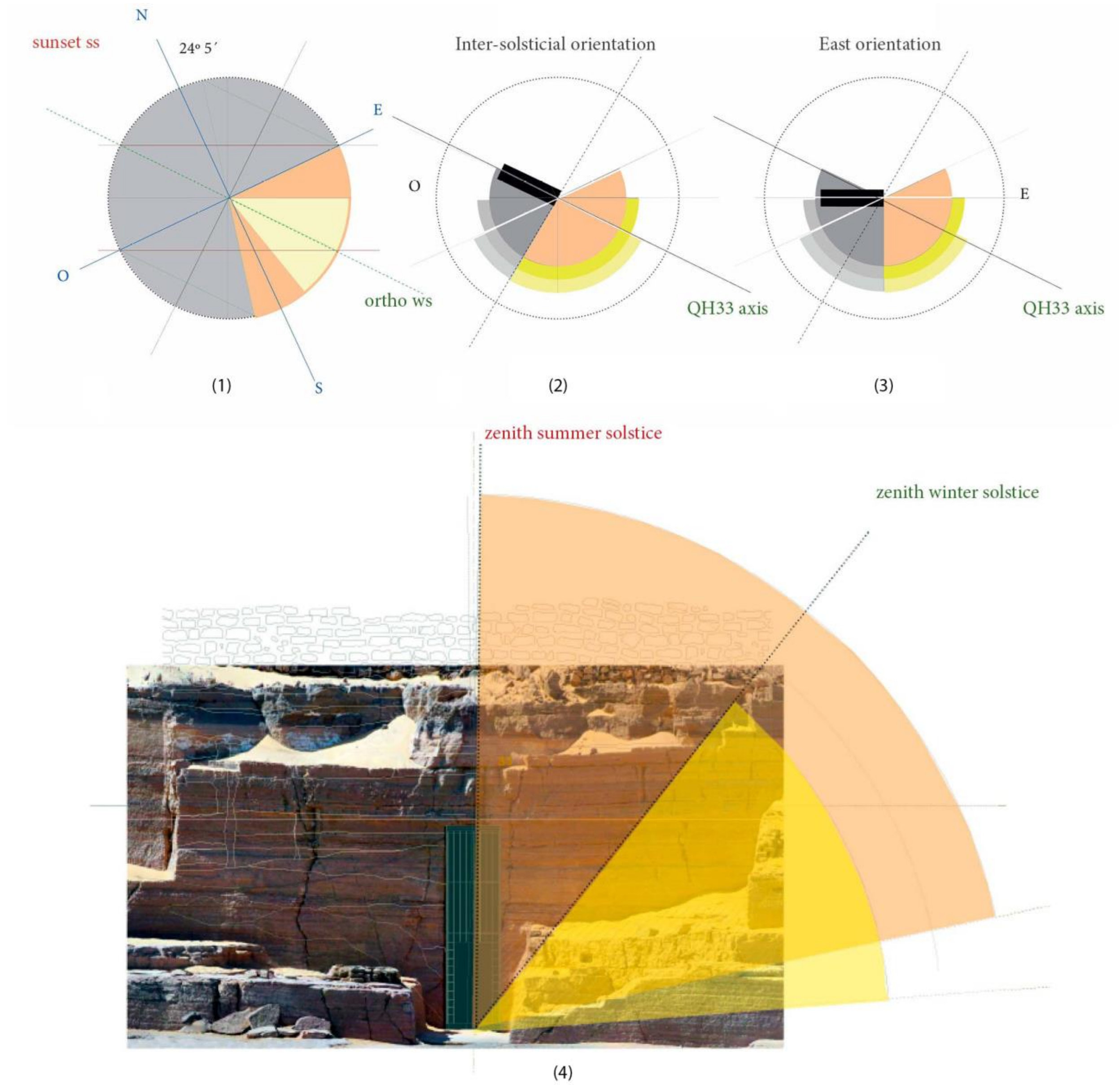

Figure 06. (1) Diagram of the ecliptic plane and intersolstice orientation. (2) Light intensity for the intersolstice orientation of QH3. (3) Luminous intensity for an orientation to the East. (4) Geometric scheme of sunlight on the façade and access door, corresponding to winter solstice and summer solstice. The graph shows the high intensity of light fallings on the plane of the façade of QH33 throughout the solar cycle.

The gateway to the hypogeum opened fully to the solar cycle, receiving direct radiation through the sun´s entire journey along the horizon to the east (52º10'). This captured the greatest amount of light toward the interior, transforming the dark space into a living space, containing the energy of the sun. The central nave of the chapel was illuminated every dawn, with greater or lesser intensity. This meant that the total length of the chapel was calculated with these conditions in mind, as reflected in the lighting results obtained. The almost cubic space for the statue of the deceased was raised on a staircase, increasing the height of the door to 4.83m, monumentalizing the space to guarantee the illumination of the niche where the Ka lived, preserving the vital breath of the deceased. The arrival of the winter solstice brought with it the rebirth of the dead, which began on the shortest day and culminated in the summer solstice, when light conquered darkness and the waters of the Nile re-emerged, fertilizing the fields after the drought.

3.2. *The horizontal plane as reference and the solar geometry derived from the right triangle*

The movements of the sun could be graphically translated onto the horizontal plane by recording the projected length and angles. The longitude reflected to the altitude of the sun in the celestial vault, while the angle indicated its position with respect to the reference point. Similarly, landscape architecture, understood as a combination of vertical and horizontal planes, considered this capacity to be the basis of geometric thought (Giedion, 2008). Each position of the sun illuminated the interior of the excavated architecture of the QH33 complex, constantly catching the sunlight. This could be interpreted as an essential resource to reflect the path of the sun, its displacement and direction on the horizontal plane, like a gnomon that casts shadows. The sun's daily passage was recorded on different planes inside QH33, based on specific location and precise orientation.

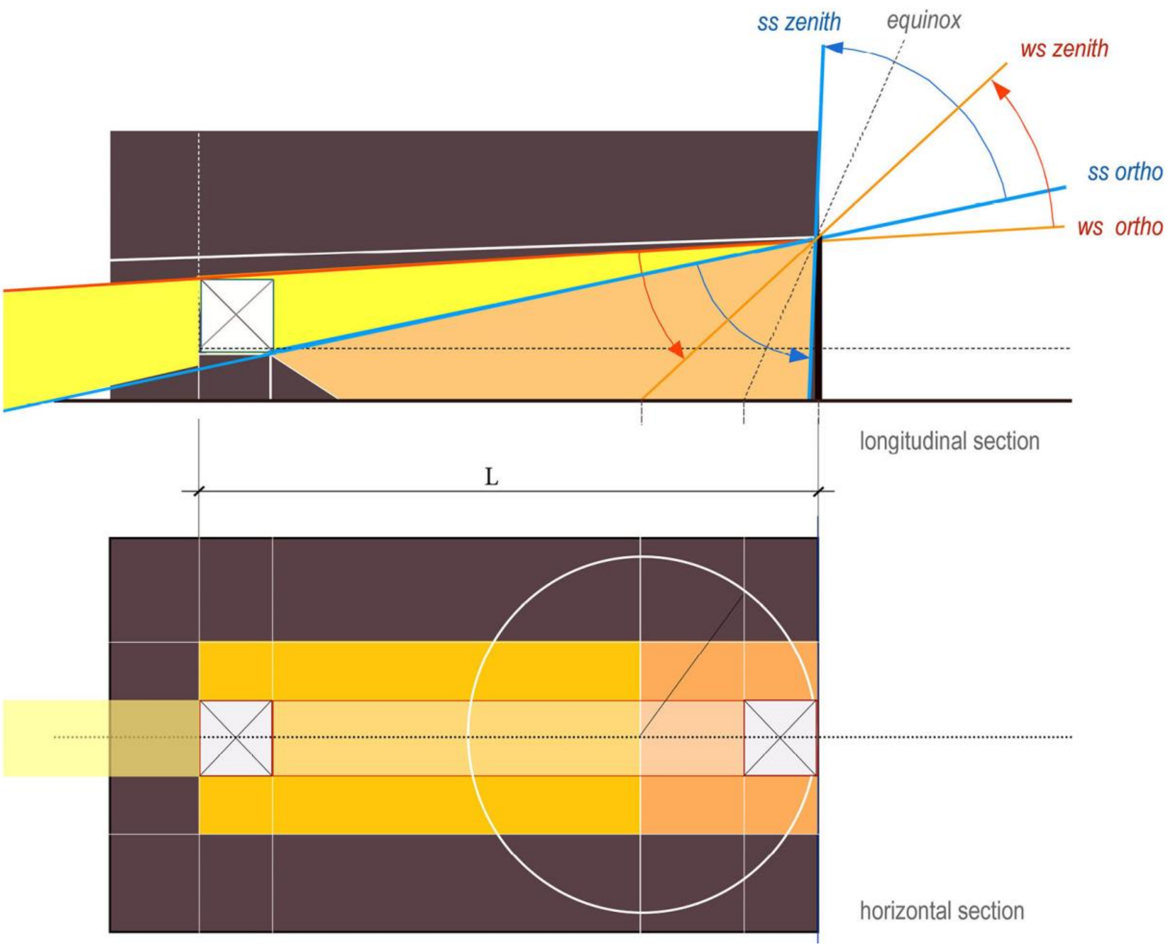

Figure 7. Plan and longitudinal section of QH33. The length (L) of the excavated space seems to be determined to achieve constant illumination of the interior space of the tomb throughout the entire solar cycle. Likewise, the height of the door is adjusted to allow the illumination of the niche-sanctuary at all solar risings from the winter solstice (WS) to the summer solstice (SS).

As has been pointed out, the singular position of Elephantine in relation to the solar cycle had a bearing on the records of its shadows, as its latitude coincided with one of the extremes of the cycle (Broda, 2000). This could be interpreted as a geographical limit, where the vertical mast never cast a shadow in a southerly direction, as the sun was almost vertical in its annual culmination. The circular shape could be seen as the projection of the celestial vault, the center of which was the vertical axis connected to the sun. The interrelation between the order of the cosmos, represented by Maat, and the horizontal plane of existence, had a geometrical reference, the approximation between the area of the square and the circle, in clear allusion to the use of the grid (Peet, 1923; Robins & Shute, 1985; Rossi, 2004) (Fig. 08). These proportions between the square and the circle could be easily translated, using a cord and square (Arnold, 1991) to draw on the horizontal plane and referring to the vertical section when conceiving the spaces. Designing with drawing was a fairly intuitive procedure that did not require complex mathematical support and could be applied in construction, as the sum of elements or parts that made up an organic unit[13]. This geometric concept was already evident in drawings and sculptures drawn on a basic grid in the form of a vertical and horizontal mesh onto which the proportions could be transferred. Divided into twelve parts with thirteen knots, it could represent the divisions of day and night. The cord would also have been used to define orientations and delimit the contour of the space to be excavated. The circumference traced by the shadows cast by the gnomon, could be divided into twelve 30º arc fragments, corresponding to the thrity-day months into which the civil calendar was organized and adjusted to the real solar cycle by adding the epagomenal days[14]. As the graph shows, using cord it was possible to draw relevant triangles such as the 3:4:5 ratios, the square (3x4) units or a double rectangle on the grid, all shapes that continually appear in architectural designs. This resulted from the gnomic growth of the diagonal square √2, which stonemasons could use to calculate slopes or to build (Lumpkin, 1980; Robins & Shute, 1985).

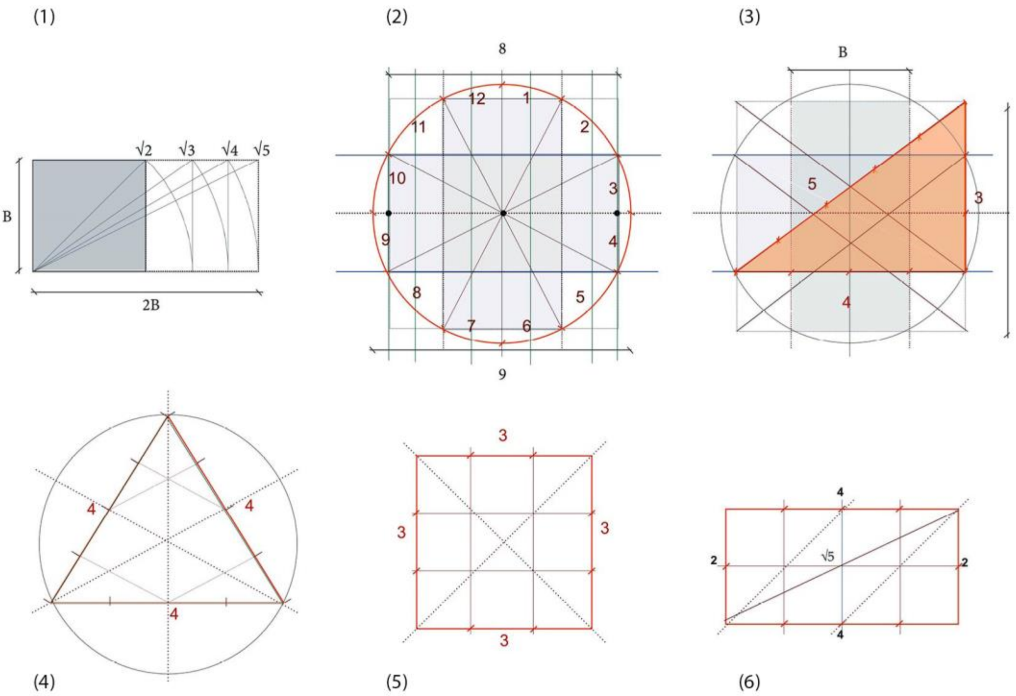

Figure 8. Basic forms used for the geometric analysis of the QH33 complex. The different shapes can be drawn using a cord as a graphic and measuring instrument, using a module (B) and its harmonic growth:
(1) Relative growth of the square of side (B), to the rectangle (B x 2 B), progressively using the diagonal as radius.
(2) Relationship between the double rectangle inscribed in the circumference divided into twelve portions of 30º. (3) Relation of the rectangle (B x 2B) and the right triangle of sides 3-4-5, drawing of a string divided into twelve parts.
(4) Equilateral triangle of 4-4-4 inscribed in the circumference, whose sides add up to 12 parts. (5) Square of side 3 x 3, traced with the same cord. (6) Double rectangle whose perimeter corresponds to the chord of 12 parts and maintains the proportion (B x 2B).

The slope or seked related the constant vertical height to horizontal displacement, in the same way that the angle of incidence of the sun was determined by the length of the shadow in relation to the fixed height of the gnomon. The access door to QH33 functioned similarly, transferring the direction, height, and movement of the sun into the architectural space[15]. Based on the analysis in section and following these geometric premises, the relationship between the space of QH33 and the translation of the sun was analyzed through the geometric analysis of its floor plan. The starting point was set during the winter solstice with the sun at its maximum decline. The right triangle traced by the sun at that time approached the diagonal √2, with a slope (1:1), decreasing as time advanced. This maximum slope of the winter solstice increased progressively until it reached the longest day, the summer solstice, at which point the slope was almost vertical (1/0). Given that the total length (L) of the space of QH33 was determined based on the longitudinal section in order to illuminate the niche, as already seen, a series of geometric relationships on the plan can explain the harmonious composition and proportions of the set (Fig. 09). The right triangle drawn for the winter solstice determines the maximum penetration of the sun at its culmination (C). If a circle is traced up to the façade plane, with the center at (C) and radius (A-C), a point of intersection (P) is obtained with the azimuth or amplitude of the summer solstice. This point served as reference when establishing the width (B) of the space to be excavated. The resulting rectangle (L x B) is a double square connected to the exterior through the door, the height of which is established for stability sand to allow the sunlight to reach the sanctuary at dawn every day. A small corridor between the door and the chapel controls the entry and distribution of light.

Taking as reference a grid of modulus (2x2) cubits, it is possible to establish an entire compositional logic for the final space and its proportions[16] (Fig. 10):

- (a) The double square (B x L) is a rectangular shape, widely used for its proportions, which could be drawn from a square with a diagonal √2 by gnomic growth until the proportions 1:2:√5 were reached. The longitudinal axis of the plan involves the progressive connection of three points, I, II and III.
- (b) Starting from extreme point III, in the sanctuary, and using the cord to transfer measurements, the telescopic right triangles with a common base could be traced, cutting the beam of light, which illuminated the central space. The cut points define a distance or module (M) that appears repeatedly in the geometric composition of the space, both horizontally and vertically (Fig. 11).
- (c) The points of intersection on the central space indicate the vertices of the square pillars that structure the space longitudinally and transversely. The 1.05m (2 cubits) grid, corresponding to the dimension of the pillars, regulate the entire plan. The resulting rectangular space is adjusted to a final length equal to √3. As will be seen later, the length of the access corridor is no accident but rather controls the projection of light at each sunrise. At the same time, spatially it requires an increase in exterior-interior tension, lengthening and narrowing the step to focus the direction toward the sanctuary at the opposite end.
- (d) The resulting horizontal composition is an ordered and articulated space, creatinges a harmonious spatial perception. The vertical and horizontal design defines a space with a rhythm set by the solar cycle. The constant lighting of the space through the door and its projection make the relationship with the times of the sun evident. Like the vertical gnomon casting shadows, the behavior of the QH33 space is based on the projection of sunlight.

4. DISCUSSION

4.1. The lighting of the architectural space in QH33 and its relationship with the solar cycle

Just the rising of the sun on the eastern horizon line could be marked daily at dawn, in the QH33 complex the projection of light from the access door to the sanctuary could also be recorded following the longitudinal axis. The orientation of the main intersolstice axis (115.24º) joined the longest night and the longest day, covering the entire path of the sun. The small deviation between the longitudinal axis (115.24º) and that perpendicular to the façade plane (118.26º) could be explained as an adjustment making use of the conjunction of both events from the sanctuary, following the carving of the star-oriented vertical façade plane. Along with the precise astronomical orientations, it should be noted that the façade plane was drawn perpendicular to the line of the maximum slope of the hill, reaching the bed of the Nile. The limit of the city built on a large island coincided with the limit of the cycle in the Northern Hemisphere, where around the summer solstice the sun remained almost vertical (around 88º) for several days. This event, known as the zenith pass (Belmonte, 2010; Martin, 2015) occurred when the vertical pole did not cast a shadow and the zenith light illuminated the depths of the oldest nilometer in the city. Thus, toward the south of Elephantine, in the equatorial strip, the zenith passage occurred twice a day, whereas it never did toward the north. In its cycle between winter solstice and summer solstice, the sun passed twice through the equinoctial point, when the sun rose from the east. At latitudes north of Elephantine, the sun at noon always cast its shadow toward the south, without ever reaching the zenith. Undoubtedly, Elephantine must have been a landmark, both in geographical terms, as it was located just before the first waterfall, and in astronomical ones. From this latitude the decans associated with the brightest stars in the Egyptian sky could be glimpsed (Clagett, 1995; Parker, 1974).

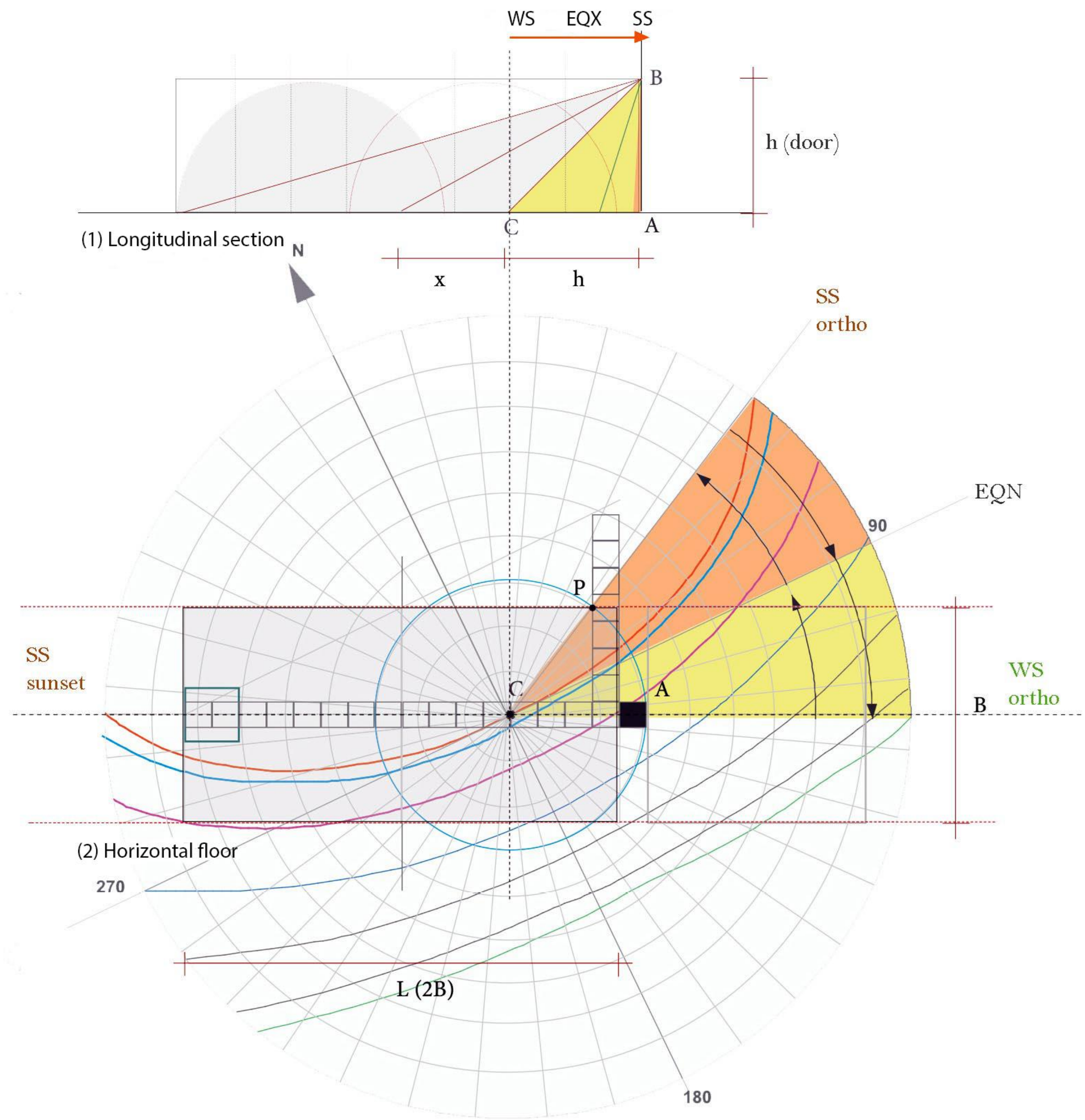

Figure 9. Geometric scheme to determine the dimension (B) of QH33. From the previously established longitude (L), together with the intersolstice orientation. (1) The height (B) of the gate allows to trace the right triangles of light inside QH33 throughout the solar cycle. (2) Triangle ABC is determined by the position of the sun at its highest elevation during the summer and winter solstices. On the horizontal plane, the azimuth angles corresponding to the two extreme positions condition the width (B) of the space to achieve maximum lighting inside the excavated space.

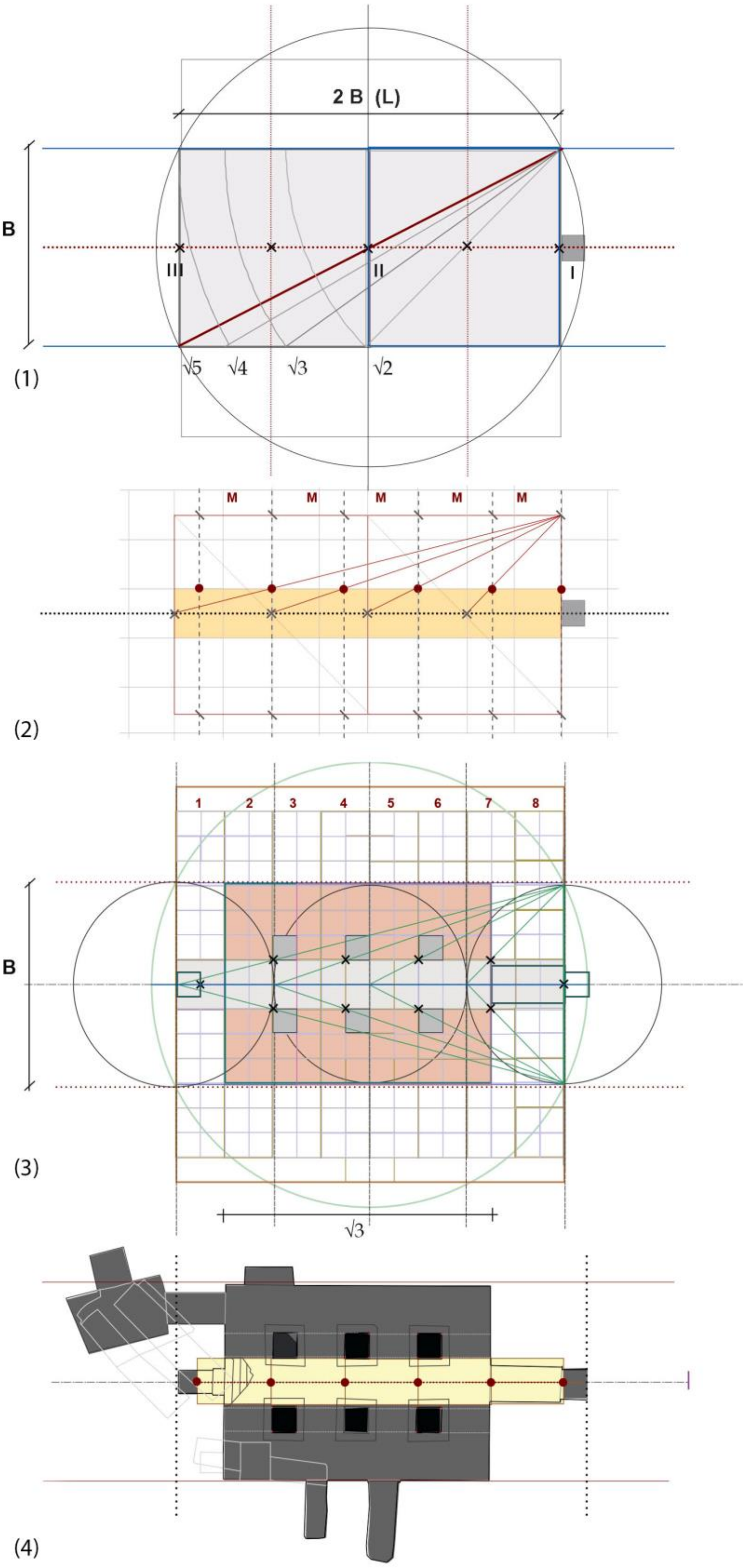

Figure 10. Geometric analysis of the plan of QH33.

(1) Double square and longitudinal axis on base grid. (2) Triangles drawn from the extreme points of side (B) towards the longitudinal axis. (3) Module (M) that determines the position of the pillars and the dimension of the access corridor. (4) final composition of the excavated space according to the geometric proportions analyzed.

The hours of sunlight were progressively lengthening and increasing in intensity, which meant that during the summer solstice, the sun remained above 30º altitude for about eight hours (Fig. 11). During this time, the sun travelled a horizontal or azimuth distance of 80º to 260º, remaining almost perpendicular to the earth's plane. In contrast, during the winter solstice, the sun slowly rose to a maximum point around 43º. In its, the sun moved horizontally between 120º and 240º. During its journey, until the culmination of the sun, the elongated projected shadow covered a wide space on the horizontal plane. As in any construction, the entry of sunlight through the access door conditioned the design of the architectural space, which was based on the derived geometric relationships. Thus, from the winter solstice, when the sun is at its lowest and in its southernmost position, space projects the light of dawn, capturing most light in the sanctuary, when the sun is moving slowly. This condition can be verified with the recorded data (Table 3), when the central part of the sanctuary is illuminated for 55 days, around the winter solstice, estimated on January 5, 1825 BC. From this point and until the equinox, the sun reaches its zenith, slowly traveling on a horizontal plane, coinciding with the start of the period of sowing and preparing crops.

When the equinox approached, at each dawn the sun ascended more quickly, with hardly any horizontal space, until it culminated in the summer solstice. The interconnections of ritual and architectural space are thus evident and can be seen in the design of its forms. The winter solstice and the lighting of the Ka of the deceased were the most relevant moment in the design of the horizontal space. From that day on, following the cycle of the sun, the statue of the deceased reactivated its vital force through the energy transmitted by the sun. Thus, the constant presence of the sun inside the tomb was a direct consequence of its studied orientation[17]. Just as the sun rose progressively until it reached the zenith, the deceased advanced until he reached the sanctuary represented by his image, once his body had been deposited in the grave (Fig. 11). Every architectural space is loaded with symbolism revealed through light and its passage. Thus, from the center of the sanctuary it was possible to see the other end until the sun reached the zenith, the limit to infinity, beginning the process of reunification of the Ka with the mummified body below. Therefore, the gateway to QH33, facing the winter solstice sunrise, and the shrine facing the summer solstice sunset, connected the longest day and night, the entire period. The path of the sun inside could be interpreted as the two stages between solstices, architecturally represented through the longitudinal axis, which connected the two extreme points. The first route, along the axis to the sanctuary, is horizontal, while the second, from the sanctuary or center, is vertical, with the sun located on the vertical axis[18]. Crossing the threshold of the tomb symbolized the beginning of the personal transformation, which the deceased had carried out during his earthly life, preparing for death. Once this life was over, he could cross the other door looking west to contemplate the sunset of each day, following in the footsteps of the god Ra. The vertical stage inside the tomb, approaching the summer solstice and the rise of Sothis (Kelley, 2011), marked the rebirth of the Nile, when the deceased began his new stage guided by Osiris. The sun passed from the equinox to the summer solstice, when it quickly rose, prolonging the hours of light and heat in the region.

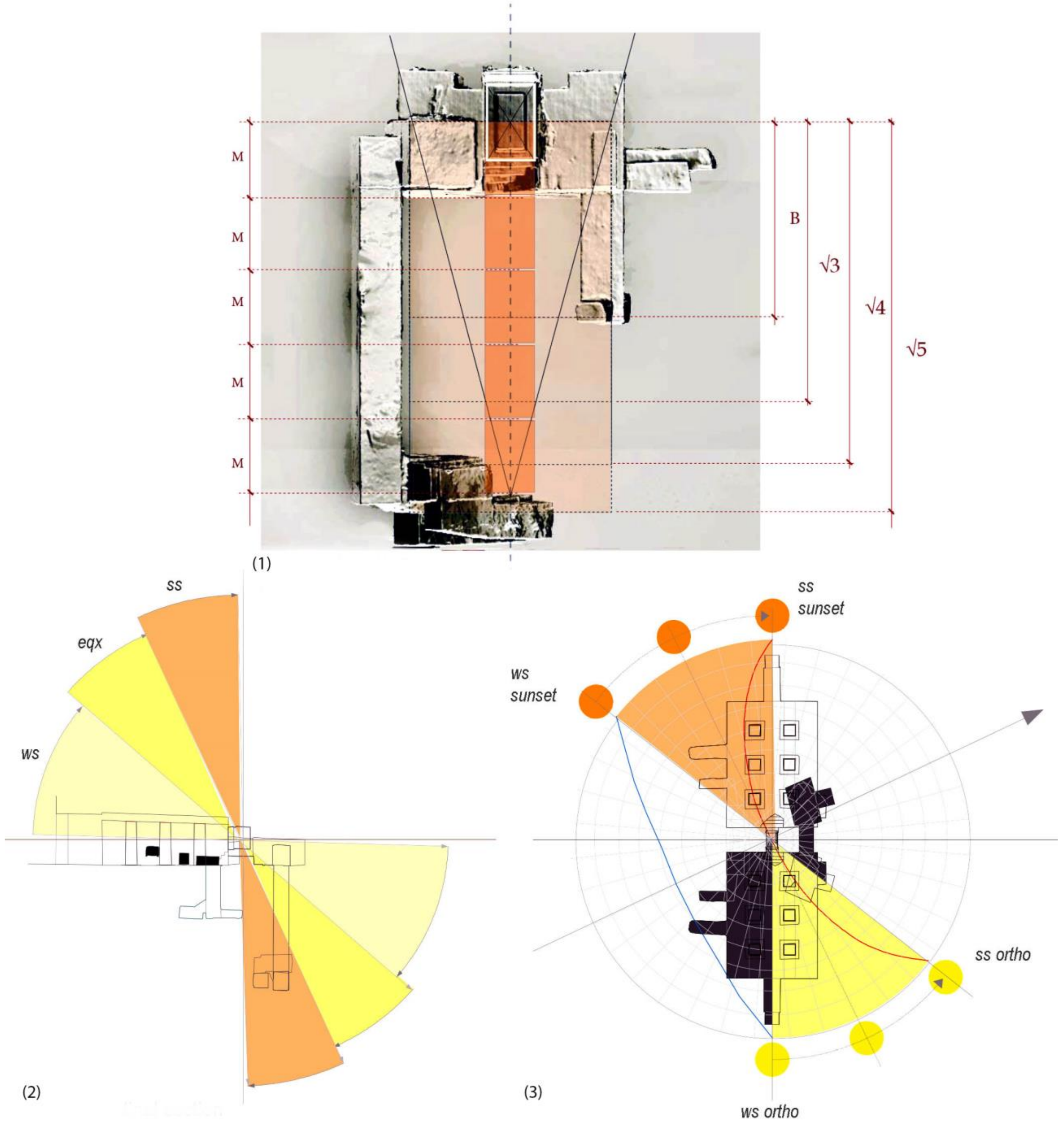

Figure 11. (1) Vertical section of QH33 through the sanctuary. The image shows the vertical alignment with the main burial chamber. The depth of the burial shaft is proportional to the module (M) considered in the floor plan. (2) longitudinal section QH33 with the different phases of the solar cycle. (3) Diagram of the azimuths of the solar cycle and its relationship with the floor plan of QH33.

4.2. The measurement of time and the composition of architectural forms

QH33 was conceived to assist the burial ritual, closely related to the sun and its cycle, from the very beginning. This link could be sustained on a geometric basis as a meeting point between the celestial order and architectural composition. The connection between the solar cycle and architecture was the result of the exhaustive knowledge of the sun cycle and its trigonometric translation, which affected both its function and its image. Following analysis of the previous Middle Kingdom burials, it can be seen that the QH33 perfected a model which adjusted its shape and composition meticulously based on orientation.

The two moments in time corresponding to the solstices were a clearly linked with the ends of the shaft, the access door, and the sanctuary. Between the two points, while the sun remained still for several days, the intersolstice cycle of six months began, to be repeated continuously. Through its apparent path, the sun travelled on the plane of the ecliptic at a rate of 1º each day (De Jong, 2007), for a total of 365.2422 days. This period was calculated approximately by measuring between two solstices and through the record of the sun on the horizon each sunrise (Fig. 12). The azimuth on the horizon moved at a rate of 30º every 30 days. When the sanctuary was illuminated at the winter solstice, the sun was progressively projected on the pillars to the south of the axis as it moved north on the horizon. The equinox, when the sun rose to the east, was a singular point in the twice-yearly pendular path of the sun. The point was considered in the architectural design of the QH33 complex, coinciding with the sunrise from the chapel after illuminating the first of the pillars. From the summer solstice, once the still sun dawned illuminating the threshold, the six-month cycle began again, marking the equinox with the illumination of the first of the pillars. As can be seen from the geometry the length of the access corridor was undoubtedly designed for this purpose.

In contrast, the space to the north of the axis was dedicated to the entrance to the chamber where the deceased's body was prepared before placing it in the niche, in a dead angle where dawn was never projected, an intensely symbolic gesture (DeYoung, 2000). The chapel also made it possible to have a wide dark space on both sides of the central corridor flanked by the pillars, awaiting the great event that was the direct lighting of the sanctuary during the winter solstice. There, relatives and priests could make periodic offerings to the deceased's Ka, awaiting the great event when the sanctuary was illuminated (Table 3) for several days during the solstice period.

The sunlight inside QH33 was undoubtedly a reference to measure time, as in any building illuminated by the sun's passage. The changes in the projection of light inside the space, its length and direction, were the two variables that determined the position of the sun throughout its entire cycle. By fixing the precise moment of sunrise, the length and direction of the light beam in the vacuum of QH33 became clear indicators of the position of the sun on the horizon, between its two extreme positions. The four-month winter period (Peret) began when sowing took place with the illumination of the sanctuary. During this stage, the sun traveled the entire space of the chapel until the equinox, when it was captured inside the entrance corridor. A period of drought (Shemu) then began, as the sun left the space of the chapel. Throughout this season, the dawn sun traveled through this narrow space, until the arrival of the summer solstice, when it barely illuminated the threshold of passage, before ascending quickly to the vertical, where it remained, dimly illuminating the almost-dark interior space. This day was the most important of the year[19], heralding the great event of the Flood, which gave way to the wet season (Akhet) after the drought. During this period, around the summer solstice, Sothis could be glimpsed on the horizon from the sanctuary after a period of invisibility, as can be seen from the alignments studied. The dimness in the chapel and the sanctuary at dawn of the summer solstice made it possible to observe the emergence of the brightest star in the sky from inside the QH33 funerary complex.

There is no doubt, that the image generated at these times was a very significant part of the ritual, and one that had implications on the composition and design of the architectural forms:

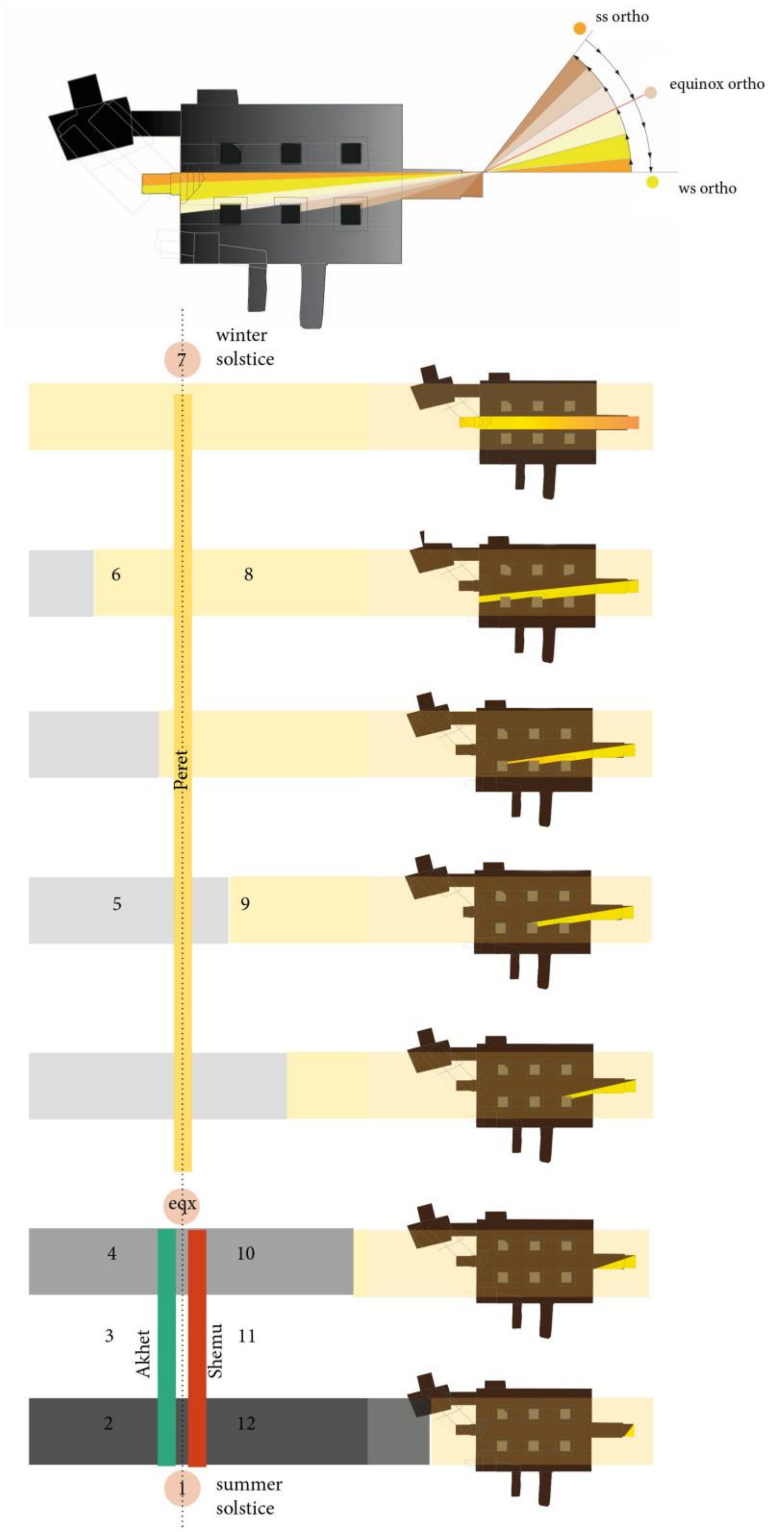

Figure 12. Illumination diagram of the interior of QH33 between the winter (7) and the summer solstice (1). During the Akhet season (1-2-3-4) the rising sun stays in the access corridor. The equinox marks the entry of the sun into the chapel, until it reaches the sanctuary directly, when the sun progressively illuminates each of the pillars. Once the sanctuary (7) is illuminated, the sun begins again to illuminate the pillars of the chapel at each dawn until it reaches the equinoctial point (10), marking the start of the season of drought or Shemu. The sowing season or Peret occurs between the points (5-6-7-8-9). The design of the length of the access corridor establishes the change of season, between Akhet, Peret and Shemu, at the equinox point, as well as measuring of each month through the transition of light inside the space.

- The longitudinal beams linking the two lines of pillars flanking the central vaulted space emphasized its depth, lengthening the perspective during the entry of light. Equally, the disappearance of the second corridor found in the two previous tombs, QH31 and QH32, was offset by the optical effect achieved by the space focusing all attention on the back wall where the sanctuary was located on an imposing staircase. This emphasized the great interior height, creating the effect of a long telescopically projected corridor in the form of steps.
- The sloped carving of the internal faces of the pillars which limit the central aisle reproduced the effect of the sun on the horizon between the pylons of the solar temples (Magli, 2013), the Akhet sign, visible when looking out from the sanctuary. This constructive feature makes up for the optical effect of the perspective, lengthening its dimension by drawing the vanishing point toward the sanctuary. At the same time, a vertical optical illusion was achieved by intensifying the perspective [20], in a clear allusion to the depths, where the primeval waters lived and where the sun sailed on the other side of the plane of existence. This is where the mortuary niche of the deceased was located, where the sun passed as night came (Fig. 13), and where the rising level of the Nile would come when the Flood arrived.
- It should be noted that, in geometric terms, the module (M) and the diagonals derived from the gnomic growth of the square used to organize the floor plan are also repeated vertically. The horizontal longitudinal axis was transformed into the vertical one as the slope of the sun increased to reach the summer solstice. As the image shows, the exhaustive constructive geometry of the complex clearly vertically aligns the mortuary niche with the center of the sanctuary (Fig. 11).

4.3. *The civil calendar and its relation to QH33*

Around the third millennium BC (3000-2750), the Wepet Renpet, the first day of the civil calendar, was established, possibly when the overflow of the Nile and the summer solstice coincided, heralded by the rising of Sothis [21]. The measurement and management of cyclical time were key to the local structure and had to be adjusted to harmonize agricultural work, coordinating with the arrival of the seasons. The vegetative cycle was linked to the observation of the stars, the phases of the moon and the daily rebirth of the sun. The calendar was a way of measuring and systematizing the passage of time to superimpose a homogeneous structure valid for any place. It considered time as a line with a beginning and end, as a succession of events arranged chronologically, while Egyptian time was considered an eternally present, and constant cycle following the fluctuations of the Nile and agricultural processes. The measurement between the two summer solstices established the 365 days of the calendar, organized into twelve months of three weeks, with ten days each, adding the five epagomenal days, to approximate the longest solar cycle, 365.2224 days. The added five days served to commemorate the gods, and the rising of Sothis was expected at the end of these (Krauss, 2006; Depuyt, 2008). Establishing a common calendar for the entire territory answered to the need to organize an economic, administrative and bureaucratic structure. This was almost the most important day of the year for a kingdom based on agriculture and guided by the Sun god.

Elephantine marked the onset of the rising of the Nile after the tropical rainy season, when it reached its maximum height, to progressively flood the northern territories. The reappearance of Sothis, with a time cycle very close to the tropical cycle became an indicator of the arrival of the summer solstice, given the irregularity in the beginning of the Flood. The brightest star in the sky, it was visible from all points of the territory, although Elephantine had been a designated place for its sighting since the end of the predynastic period. The appearance of one of the brightest stars in the sky became an omen of the arrival of the Flood, visible from all over Egypt, although its proximity to the solstice depended on the place of observation and the precession of the equinoxes. Just as the Flood was gradually delayed in a northerly direction and the sighting of Sothis was a reference, so the civil calendar varied according to the seasons, at the rate of one day every four years. Due to the lag between the natural cycle and the civil calendar, which began in Middle Kingdom (2000 BC), the New Year coincided with the winter solstice. This lasted for more than sixty years, and explains the precise orientation of the winter solstice of QH33 and the other hypogea. Due to its proximity to the first waterfall and the unique perception of the summer solstice, the day of the gnomon without shadow (Spalinger, 1994) or the zenith pass (Martin, 2015), Elephantine became a geographical and territorial reference point for the control of the natural cycle of the seasons, irrespective of the lack of coordination with the imposed civil time. As we have seen the link of this territory with the sun and the rebirth of the Nile corresponded to the limit of the intertropical zone. Both events, the rising of the Nile and the day without shadow, connected the geographical and celestial planes of existence and were simultaneously perceived in this geographical place, which became a point of reference. It seemed logical to establish this limit assumed by the First Nome of Upper Egypt as a zero point[19] (Fig. 05). Imposing a target time for the entire territory, regardless of location, involved a simplification derived from geometry, establishing a total of 12 months, in each of which the sun moved about 30º, with five days added to complete the process. The small misalignment of 0.2224 days could be replaced with a geometric register, observing the arrival of Sirius, whose cycle was very close to that of the sun, 365.2510 days, so that both measurements were used at the same time, a wandering calendar (Lull, 2016).

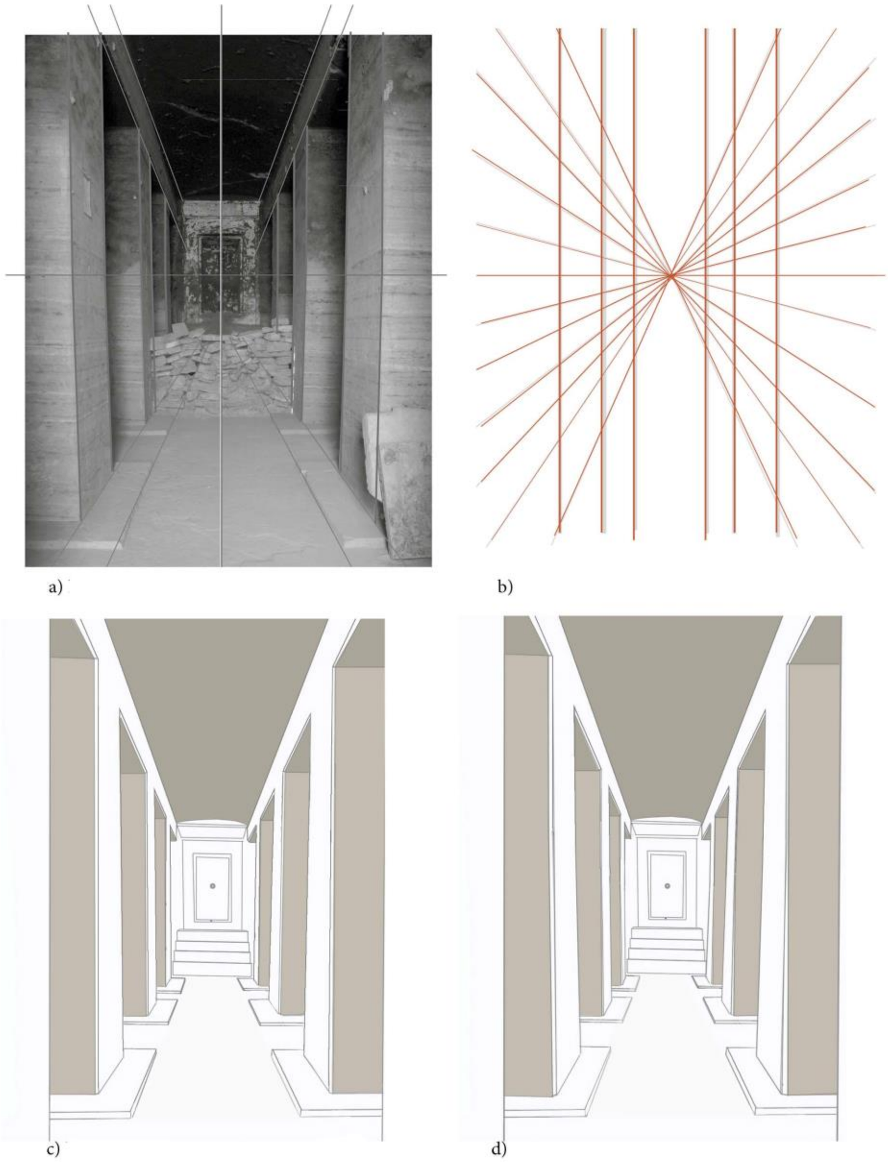

Fig. 13 (a) Perspective image of the chapel of the QH33 complex. (b) Graphical representation of the Hering Illusion. (c) Geometric drawing of the interior perspective with the lines of the pillars rectified vertically (d) real geometric drawing of the interior of the chapel with the carved pillars. The sloping internal face of the pillars enhances the optical effect of perspective to indicate the idea of depth, both towards the niche and towards the funerary well.

It was impossible to keep civil time synchronized with solar time, so that the beginning of the year shifted through the three seasons. The relevant dates of the solar cycle were ahead of the established pattern, and were not to coincide again for another 1,461 years. The mismatch that occurred every four years with respect to the sun could be compensated through architecture, capable of projecting shadows (pyramids) and recording the light inside, constantly and permanently. Thus, perfectly located and oriented, a construction like QH33 could trace an axis as a reference in measuring the solar cycle, similar to that of a compass in space (Belmonte & Shaltout, 2009; Belmonte 2009). The most important constructions, temples and tombs, fulfilled this function in each location, and this explains their orientation along the valley. A place designed and transformed by the agency of a community and its architecture could also be a reference on a larger scale within the landscape (Ingold, 1993).

5. DISCUSSION AND CONCLUSION

The image (Fig. 14) graphically shows the conclusions of the vertical and horizontal analysis carried out on QH33. By recording the sunlight inside, the tomb functioned like a vertical mast, capturing the sunlight inside the dark chamber. The height of the door, designed to achieve the daily illumination of the sanctuary at dawn, worked like a gnomon, and acted as a light clock. Architecture worked by measuring the space traveled by light in a particular time interval, specific to each location of architecture in the landscape.

As shown in the graph, starting from the summer solstice, the light barely illuminated the threshold of passage, during approximately the thirty days of the first month of the Flood. During this interval, months 1 to 3, the sunrise sun was inside the lantern or transition hall toward the main space of the chapel. In this season of maximum intensity, the sun in the zenith marked the longest days of the cycle. Between months 4 and 5 the equinox took place, just when the rising of the sun began to illuminate the first of the pillars to the south of the chapel. From here, the drought and harvesting season began during months 5 and 6 when the sun was progressively projected on the three pillars on the left, until it reached the sanctuary. In months 6 to 8 the winter solstice marked a turning point in the cycle, remaining fully or partially illuminated (Table 3). At the end of month 8, the sunlight had left the sanctuary and again began to illuminate the pillars from the inside toward the door during month 9. Again, the equinox in month 10 marked the entry of the sun in the access corridor to the door, announcing the arrival of the summer solstice in month 12. The geometric analysis shows that during intervals 5-6 and 8-9 it ran the entire length of the chapel, marking each of the three weeks and their days on the horizontal line. This horizontally control of the three-week duration of the cycle was used to organize and manage sowing. During the time in which the sanctuary was illuminated, the space travelled by the light on the horizontal plane was 2.10m, as shown in the dimensions. This measurement was the reference module repeated as a constant in the geometric composition of the set, and was equivalent to 4 royal cubits, corresponding to the reference grid considered for the analysis of space in its vertical and horizontal planes.

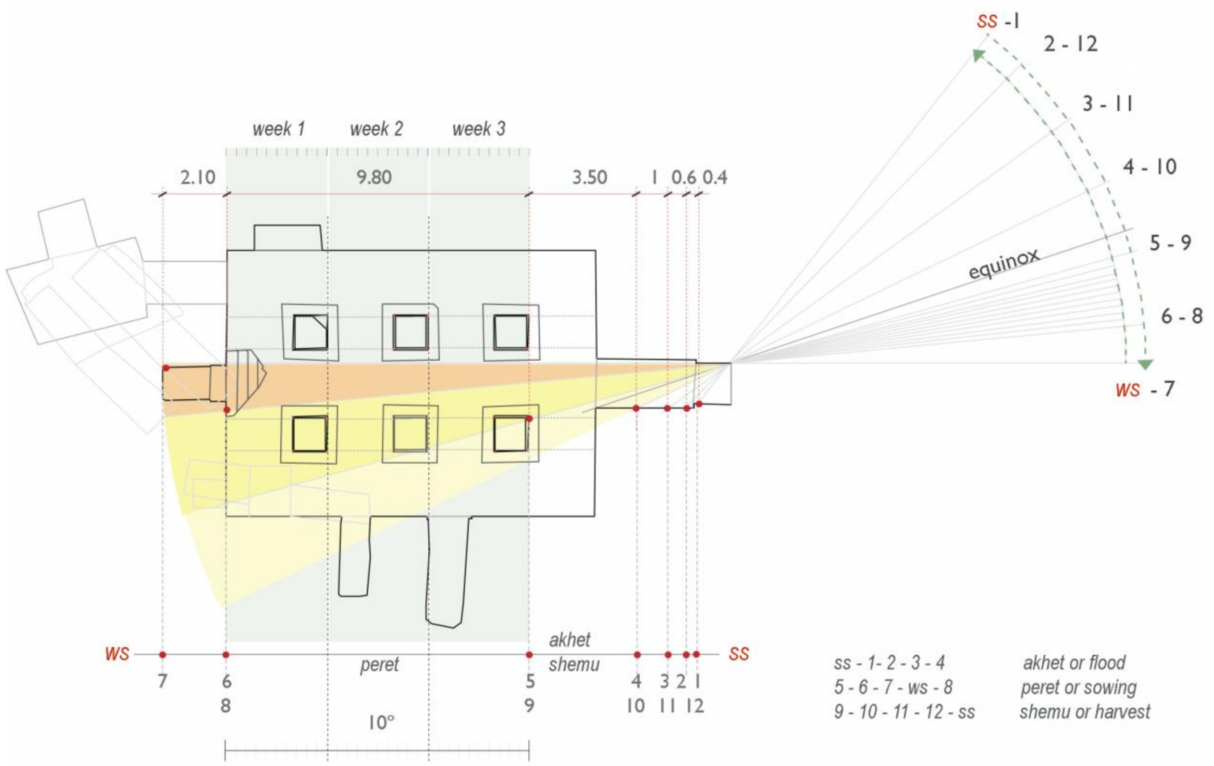

Fig. 14 Geometric diagram of the path of sunlight at each sunrise between the winter (WS) and summer (SS) solstice. The numbers indicate each of the months of the three seasons, Akhet, Shemu and Peret. The upper horizontal line marks the 3 weeks that make up the two months before and after the winter solstice, during the sowing season or Peret.

With the results obtained from the QH33 lighting simulation and based on the geometric proportions analyzed, we can conclude that:

- The sun, a symbol of rebirth was the common thread of the QH33 project, linked to its geographical position (flood) and its latitude, and coinciding with one of the poles of the solar cycle and maximum incidence of the sun.
- Using triangulation to reference the position of the sun with respect to a specific point through the projected shadows established a unique relationship between the sun and a geographical position. This relationship could be traced through the right angle triangles corresponding to the solstices and equinox, as singular moments of the cycle.
- Elephantine, as a predynastic settlement, from where the progressive rise of the Nile toward the north was controlled, may have been a geographical reference point for the establishment of the first day of the civil year, when the summer solstice coincided with the rise of Sothis.
- Architectural spatial thinking was based on the composition of vertical and horizontal planes. The analysis of the lighting in QH33 complex according to this geometric scheme has allowed us to interrelate its design with the position of the sun throughout its complete cycle. The architectural design of QH33 records the different phases of the solar cycle, making it possible to measure these regardless of the established calendar dates.
- Some of the formal and spatial qualities analyzed in QH33, as well as its functionality and meaning, are intimately connected with the sun. This relationship and its geometric translation support the beauty and harmony of an ensemble that can be considered as the culmination of the evolution of this unique type of architecture in the Middle Kingdom of Upper Egypt.
- The exhaustive orientation and constructive rigor of QH33 show the architects' extensive knowledge of the solar cycle and its application to the project supports the hypothesis of a funeral ritual linked to the Sun.

The QH33 architectural complex can be considered the oldest private funeral project in which a perfect intersolstice alignment is achieved, showing the presence of individuals with enormously sophisticated religious and technical training in provincial courts such as Elephantine.

AUTHOR CONTRIBUTIONS

Conceptualization, methodology, investigation, writing: Joyanes Díaz, Martínez De Dios, Jiménez Serrano, Muñoz González, Ruíz Jaramillo; software, validation: Mozas Calvache; formal analysis: Joyanes Díaz; supervision: Jiménez Serrano; funding acquisition: Joyanes Díaz, Jiménez Serrano. All authors have read and agreed to the published version of the manuscript.

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NOTES

1 Toward 5000 BC the angle of the plane of the ecliptic was 24º 22´, whereas in 3700 BC it stood at 24ºº18´. Around 3000 BC the maximum declination of the sun was 24ºº 05´, coinciding with the latitude of Aswan and Elephantine. This inclination progressively decreased until it approached the latitude of Elephantine 24º08 '. At this southern limit, the sun was at its highest point almost verticall. The change in solar declination due to the progressive modification of the obliquity of the ecliptic is about 0.46´´per year, which is equivalent to 0.5ºº every 3900 years (Belmonte, 2006; Belmonte, 2009).

2 The total number of hours of sunlight throughout the day increased progressively from the winter solstice, reaching its maximum angle at the summer solstice. The slope of the right triangle, formed by the shadow and the gnomon at the maximum declination of the sun, during the winter solstice, increased every day, until it was very close to the vertical axis, the zenith. As a result of this geometry of the solar cycle between the solstices throughout the year there was a great incidence of sunlight on this location (Robins, 1995).

3 Elephantine was a landmark for observing Sothis, when the star appeared before dawn after about 70 days of invisibility. This essential to marking the first day of the first month, the New Year, as well as to adjusting the 365-day calendar (Belmonte, 2006; Shaw, 2003; Lull, 2016).

4 The rise of the star Sothis was first observed from Elephantine. It was sighte from Thebes about two days later and from Memphis about six days later. This difference was due to the inclination of the ecliptic, as the star was visible earlier from further south. The distance between Elephantine and the coast, with a difference of approximately 7º of latitude meant a difference of about 28 years in the Sothic cycle (Wells, 1985; Kraus, 2006).

5 The rise of Sirius, after seventy days of invisibility, was decisive in the measurement of the solar year. The brightest star appeared just before sunrise, marking the beginning of the flood season. The star identified with to the goddess Satet reappeared to herald the agricultural cycle. Around 2750 BC the sighting of the star occurred firts in the extreme south of the area, at 24ºº latitude, and took place on June 16, according to the Gregorian calendar (Wells, 1985; Wells, 1994; Krauss, 2006).

6 Stellarium is a free, open source planetarium that displays a realistic 3D sky, as see it with the naked eye, binoculars, or a telescope (http://stellarium.org/).

7 For example, on July 17, for a Sirius elevation angle of 9º, its azimuth was 115.3º, considering a solar elevation of less than -6º.

8 The cult of the sun was related to the origin of the sacred benben stone, which was illuminated by the sun after emerging from the primordial waters. The gnomon was a similar image, a vertical pillar that served as an instrument for analyzing the geometry described by the sun in its annual path. The complete cycle of the sun, its movements and the relevant moments for a specific geographical location could be ascertained from the projected shadows (Kelley & Milone, 2011; Rossi , 2004).

9 The recording of the direction of the shadows made it possible to measure the angle of incidence of the sun throughout the day. It was necessary to know the height of the sun and consider the effect of latitude to divide the day into 12 parts. The relationship between the

shadows of the shortest day and the longest day, the solstices, was an indicator of the geographical position from which the measurement was made. With a center in the gnomon, a circle could be drawn cutting through the shadows and drawing an angle, whose the bisector indicated the Celestial North. This geometry was essential for the orientation of buildings and must have been known by astronomers-architects (Kelley, 2011; Wright, 2009; DeYoung, 2000).

[10] The celestial pole was also determined by observing the imperishable stars, which never set and always moved around this point. This celestial point could be seen from the temple dedicated to Satet in Elephantine, True North was visible from the island, due to its geographical location (Krupp, 2000; Magli, 2013; Wells, 1985).

[11] The conception of architecture was similar to a composition of vertical and horizontal planes that produced space. Thus, the excavated architecture reveaedl a eliminating strata of stone so that the light made it visible and functional (Giedion, 1967).

[12] The relationship between the triangles defined by shadow projections in the winter solstice and the summer solstice was linked to a geometric value (gnomic factor) in a geographical location. Solar geometry may have been the origin of the triangle with sides 3:4:5, the location of ancient Memphis, latitude 29°°51 (Pérez, 2007; Lull, 2016; Badawy, 1965).

[13] Knotted graded rope was used to transfer measurements and angles and to draw simple geometric shapes, such as the reference right triangle with 12 parts (3:4:5) or other geometric shapes inscribed in the circle with the grid as reference. Isosceles triangles with ratios 5:4, 5:2, 6:7 or 7:8 are common in architectural design and rectangular spaces with ratios 5:8 known as the golden rectangle and the double square of the √5 diagonal (Badawy, 1965; Ghyka, 1977).

[14] The annual journey of the sun passed through twelve constellations in a celestial belt parallel to the ecliptic, visible from Elephantine. Each group of stars travelled for approximately 30 days and was associated with the different cycles of the sun and the seasons (Kelley & Milone, 2011).

[15] Like the gnomon, the architectural space of QH33 was linked to the solar cycle, bringing light indoors to; a sacred space linked to the vital force of the cosmos that represented the sun (Belmonte, 2009; Krupp, 1998; Selin, 2012).

[16] According sources the height of the gnomon was equivalent to 4 royal cubits, approximately 2.10 m. The gnomon was used to calculate the zenith pass in the latitudes, where the sun reached the position on a perpendicular axis since pre-dynastic times. The interval between two days without shade was 365 days. Not surprisingly, the concept of the year is represented by a scepter with a curved tip in hieroglyphic writing, in clear reference to this instrument (Martin, 2015; Xiaochun, 2000).

[17] The 117° azimuth direction was perpendicular to the course of the Nile at Thebes, joining the rising of the winter solstice and setting of the summer solstice. It was a unique conjunction between the geographical landscape and the celestial landscape. This same orientation scheme was repeated in the orientation of the Satet temple during the reigns of Mentuhotep II and Senwosret II, referenced in the design of the QH33 complex. When the primitive sanctuary was built, around 3200 BC, Sirius was visible at an altitude of about 4°° and almost coincided with the orientation of the winter solstice, resulting in a double solar and stellar orientation in this territory (Shaltout, 2005).

[18] Following the journey of the sun in its different phases, meant linking to the daily rebirth, in the same way that Osiris came back to life. Symbolically placed in the niche, the Ka could look at the complete cycle of the sun once he transited to the other side, to the underworld. Receiving the energy of the sun would allow the deceased to renew himself, thus at the zenith the summer solstice, the vertical sun reached the mortuary well, to bring him back to life (Allen, 1989; Hornung, 1996; Isler, 2001; Snodgrass, 2018).

[19] From the data obtained, the summer solstice for the year 1825 BC occurred on July 14, while the summer solstice occurred on January 5. It is estimated that the calendar could have been established between 3285 and 3000 BC. Considering the real solar cycle, the conjunction of the first day of the calendar with the summer solstice would have occurred when the construction of the QH33 complex was completed.

[20] The sloping inner face of the pillars of the chapel of QH33 represented the symbol of the akhet, the image of the sun among the mountains. The perspective of the central space generated an optical effect, the Hering Illusion, causing the vertical lines of the pillars to curve. This effect was further enhanced by the constructive detail in the pillars. It showed the perspective towards the interior of the earth as an inverted pyramid where the mortuary niche with the mummified body of the deceased was located.

[21] The sight of the star was used to adjust and synchronize the civil calendar with the solar year. After this event, it was several days before the star was sighted in the north. Therefore, Elephantine could have been a reference point around 2750 BC in the southern limit of Egypt, the mythical origin of the Nile. Sirius was related to Isis, the goddess who made Osiris reborn after his death, thus announcing the rebirth of the Nile (De Jong, 2007; Krauss, 2006).

[22] The proximity to the equatorial zone would explain the worship of the solar god Khnum or Chemnu, who together with Satet and Anukis made up the triad linked to Elephantine. The cult of the Sun was also associated with the force that regenerated the Nile to fertilize the earth and with the arrival of the day with most hours of light per year. These events, along with the rising of the star Sirius, were vital in determining the first day of the Civil Year. The constellation of Orion was identified with Osiris and the Great Bear with Seth, while the star Sirius represented Isis. The stars were used to measure time from the 11th and 12th Dynasties, establishing a sequence of 12 stars or constellations, known as decans, in a strip of the sky visible from Elephantine (Quack ,2018).