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ARCHAEOASTRONOMICAL REFRACTION RECONSIDERED

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ABSTRACT

Most investigations of refraction for archaeoastronomical research have been based on observations over flat terrain (Sampson 1994) or over a depressed horizon (Schaefer & Liller 1990). Such observations are not representative of the elevated distant foresights found in most archaeoastronomical sites. It has been shown (e.g. Young 2004) that the magnitude and variability of refraction is exceptionally strong near the horizon and decreases rapidly with increasing angular altitude.

A recent study of refraction in archaeoastronomical contexts has demonstrated from meteorological principles the influence of strong temperature gradients near the earth's surface, the importance of local topography on refraction for low lines of sight, and from geometrical analysis the importance of refraction near the distant horizon marker and the relative unimportance of refraction near the observer. Quantitative investigation of this phenomenon shows that the magnitude of refraction at elevated horizon markers is reduced by one to two orders of magnitude in comparison to refraction over flat terrain (McCluskey 2017).

The current investigation extends the previous study to consider geometric factors as they relate to the calculation of refraction, the variability of atmospheric refraction, and the relative importance of different meteorological parameters. This investigation closes by discussing the methods of computing refraction that are appropriate to various levels of precision.

The results of this investigation and of prior work (McCluskey 2017) account for the consistency inferred from ethnographic reports of Puebloan astronomical observations and call for a positive reevaluation of the possible precision of archaeoastronomical alignments to distant horizon markers.

KEYWORDS: methodology, archaeoastronomical sites, observational consistency, astronomical refraction, terrestrial refraction, temperature gradient

1. INTRODUCTION

Most published studies of archaeoastronomical refraction thus far have been empirical in nature. This discussion, in contrast, approaches the problem primarily from a theoretical perspective. Due to limited space, I will omit some details which can be found in (McCluskey 2017).

I was drawn to study archaeoastronomical refraction by the well-known inconsistency between the results of ethnoastronomical research (McCluskey, 1990; Zeilik, 1985), which seemed to indicate that simple naked eye observations could yield consistent observations of the rising of the Sun and Moon, and modern empirical studies of sunrise and sunset over flat terrain (Schaefer and Liller, 1990; Sampson et al, 2003), which seemed to indicate that the variability of such observations made consistent archaeoastronomical observations impossible.

As I began considering how refraction influenced archaeoastronomical observations to distant horizon markers, I was struck by the interactions of geometry and meteorology in both astronomical and terrestrial refraction. Further investigation of archaeoastronomical refraction revealed the important role of local topography in shaping these interactions. These three local factors: geometry, meteorology, and topography became the central elements for my investigation. To anticipate my conclusion, I found that "observations over a level horizon obtain near-surface refraction one to two orders of magnitude greater than refraction over elevated horizon markers" (McCluskey 2017).

Let me open this discussion with a few general considerations. For typical archaeoastronomical and ethnoastronomical observations, such as horizon calendars, we are only concerned with the consistency of observations possible at a claimed site. For example, how consistently does a horizon marker indicate the time to plant a particular crop, a date which is not fixed by astronomical theory but by local meteorological circumstances? We only become concerned with absolute accuracy when we can relate possible observations to modern astronomical theory, as when a marker is claimed to indicate a solstice or lunar standstill.

2. GEOMETRICAL CONSIDERATIONS

To examine how geometry influences refraction, we can consider (Figure 1) an observer at *O*, watching an astronomical body *S*, as it rises or sets beyond a horizon marker *M*, along the refracted ray *OMS*.

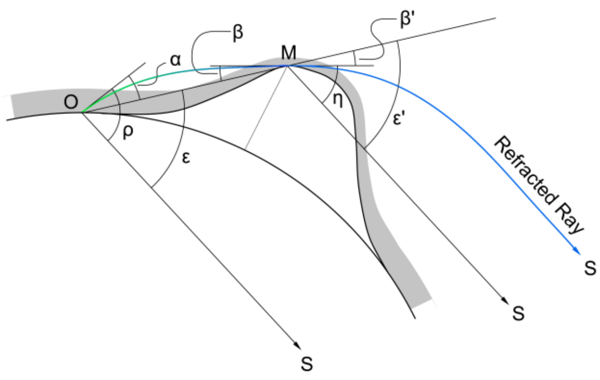

Figure 1. Refraction over an elevated horizon marker; surface layer shown in grey. Note that ε = ε' and β = β'

It has been traditional to distinguish the problem of astronomical refraction, where the observer is within the atmosphere and the target is outside it, from terrestrial refraction, where the observer and target are both within the atmosphere (Hirt et al, 2010; Young 2002-12). In this study it is useful to consider three different angular measures of refraction: The first two are these traditional measures[1].

- **Astronomical refraction ρ** measures how much lower an astronomical body *S* actually is than it appears to be. Its magnitude and precision measure the accuracy and consistency with which an observer at *O* measures the position of an astronomical body *S* and it is produced by the curvature of the entire refracted ray *OMS*.
- **Terrestrial refraction α** measures how much lower (or higher) a distant marker *M* actually is than it appears to be. Its magnitude and precision measure the accuracy and consistency with which an observer at *O* measures the position of the marker *M* and it is produced by the curvature of the portion of the refracted ray near *O*.

By analogy to those two, McCluskey (2017) proposed archaeoastronomical refraction as measuring an angle of direct concern for our investigations.

- **Archaeoastronomical refraction ε** measures how much an astronomical body *S* actually is below a horizon marker *M*, when they both appear to be at the same altitude on the horizon.[2] Its magnitude and precision measure the

[1] "'Astronomical refraction' is the angular displacement of astronomical objects from their true or geometrical position, because of the bending of rays in the Earth's atmosphere. It is contrasted with 'terrestrial refraction,' which is the corresponding angular displacement of objects on the Earth and in its atmosphere " (Young 2002-12)

[2] Archaeoastronomical refraction is similar to parallactic refraction, which relates the positions of nearby satellites

accuracy and consistency with which the line *OM* indicates the position of an astronomical body *S* and it is produced by the curvature of the portion of the refracted ray near and beyond *M*.

These three angles can be related by the mathematical relationship that archaeoastronomical refraction at *O* is equal to the astronomical refraction less the terrestrial refraction.

$$\varepsilon = \rho - \alpha \quad (1)$$

Note that since the terrestrial refraction α does not contribute to the archaeoastronomical refraction, we can see that refraction near the observer has little effect on archaeoastronomical refraction (cf. Bomford 1980, 236).

We get a different perspective on this same case of archaeoastronomical refraction by imagining the circumstances for a hypothetical observer at the horizon marker, *M*. We find that the archaeoastronomical refraction at *O* can be expressed as the sum of the astronomical refraction for the hypothetical observer at *M* and an angle equal to the terrestrial refraction β which the observer at *M* would see looking back to the original observing site, *O*.

$$\varepsilon' = \eta + \beta' \quad (2)$$

Note that the portion of the ray *OMS* near and beyond the marker *M* contributes almost entirely to archaeoastronomical refraction.

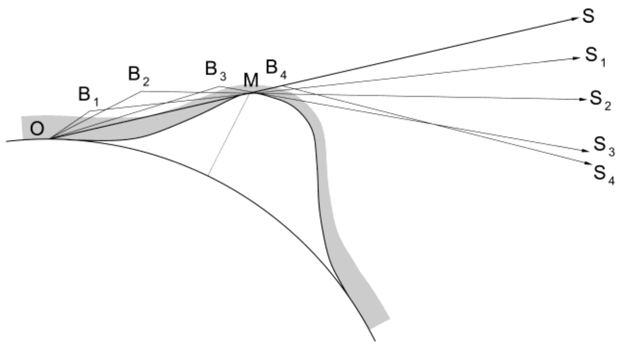

Figure 2 Bending of the line of sight Simplified model; bending exaggerated. Corresponding positions of B and S are marked by corresponding subscripts

To further explore the effect of the geometrical configuration on archaeoastronomical refraction, we can consider a simplified model (Figure 2) in which the line of sight only bends at a single point. We will assume a fixed observer at *O* and a marker at *M*, with a light path passing from *O* through *M* to a point *S* on the celestial sphere; in this model *S* will move as the refraction changes.

In this model we will assume that all bending of the line of sight takes place at a single point *B*, located somewhere along the path *OMS*. The angle of the bending at *B* is assumed to have a constant value. For the diagram we assume an exaggerated bending of about 30° at *B*.

In all cases the apparent position of the marker M changes by the amount of the bending. The point on the celestial sphere *S* which, to an observer at *O* seems to be directly beyond the marker *M*, changes by a greater amount as the place where bending occurs moves from *O* to *M* and beyond.

When the bending takes place near the observer, as at B_1, the position of S_1 differs from the position of *S* by a very small angle compared to the bending of the ray.

When the bending takes place midway between the observer and the marker, as at B_2, the position of S_2 differs from the position of *S* by an angle half as great as the bending of the ray.

When the bending takes place near the marker, as at B_3, the position of S_3 differs from the position of *S* by an angle almost as great as the bending of the ray.

When the bending takes place beyond the marker, as at B_4, the position of S_4 differs from the position of *S* by an angle equal to the bending of the ray.

Therefore, an equal bending at *B* will have an increasing effect on archaeoastronomical refraction as *B* moves from the observer *O* to the marker *M*, reaching a maximum at *M*, the effect remaining constant in the region beyond *M*.

This simplified bending model ignores two facts. The first is that refraction of the line of sight does not take place at a single point but occurs as a smooth curve, the summation of a series of small bendings at various points along the line of sight. The second is that refraction is not uniform along the line of sight but is greater when the line of sight passes through regions of greater density gradient, such as those commonly found near the earth's surface.

This simplified model can be extended by using numerical integration to sum the individual small bends produced by meteorological conditions, using parameters taken from a measured, standard, or hypothetical meteorological profile at each point along that portion of the line of sight *OMS* that is within the atmosphere.

Following Bomford's discussion of terrestrial refraction (1980, 233-236), we can use his approximate formula for the curvature of a ray $1/\sigma$, in arcseconds per meter, which Bomford claims is accurate to 2%.

or balloon-borne beacons to the background of distant stars (Kakkuri and Ojanen 1979; Bomford 1980, 517-18).

$$\frac{1}{\sigma} = 16.3\ \frac{P}{T^2}\left(0.0342 + \frac{dT}{dH}\right)\cos\theta, \qquad (3)$$

where P is pressure in millibars, T temperature in degrees Kelvin, dT/dH temperature gradient, and the small angle θ is the local inclination of the ray to the earth's surface, at each point along the line of sight.

Integrating the curvature of the ray can generate the different angles of terrestrial refraction at the observer O and at the marker M (Figure 1). The angle α of terrestrial refraction at O, which contributes to the astronomical refraction, but not to archaeoastronomical refraction, is given by the integral

$$\alpha = \frac{1}{L}\int_0^L \frac{L-l}{\sigma}\,dl, \qquad (4)$$

where L is the ray path OM from the observer to the marker, and l is the distance along the ray from the observer to the point under consideration (Bomford 1980, 233-5, Figure3.11; Thom 1958). Similarly, the angle β of terrestrial refraction at M, which contributes to astronomical and archaeoastronomical refraction, but not to terrestrial refraction at O, is given by

$$\beta = \frac{1}{L}\int_0^L \frac{l}{\sigma}\,dl. \qquad (5)$$

By considering the factors $(L\text{-}l)\,/\,L$ in equation 4 and $l\,/\,L$ in equation 5, we again see that refraction near the marker M has little effect on α and refraction near the observer O has little effect on β.

We can combine equations 4 and 5 for the terrestrial refraction angles α and β to obtain the total curvature of the ray between the observer O and the marker M

$$\alpha + \beta = \frac{1}{L}\int_0^L \frac{L-\ell}{\sigma}\,d\ell + \frac{1}{L}\int_0^L \frac{\ell}{\sigma}\,d\ell = \int_0^L \frac{1}{\sigma}\,d\ell. \qquad (6)$$

This path integral of the curvature can be extended to the limits of the atmosphere to compute the astronomical refraction ρ at the observer O

$$\rho = \int_0^U \frac{1}{\sigma}\,dl, \qquad (7)$$

where U is the distance from the observer O at which the ray reaches the height of the upper limit of the atmosphere.

Alternatively, we could adapt one of the standard integration models for computing astronomical refraction (Auer and Standish, 2000; Hohenkerk and Sinclair, 1985; van der Werf, 2003). Since the surface layer follows local topography (Figure 1), the density of the atmosphere is not spherically symmetric and the integration cannot assume that the refraction invariant (nR sin z) is constant along the ray (Young 2006). For a comparison of integration models see van der Werf (2008).

3. METEOROLOGICAL FACTORS

It has long been recognized that density gradients in the atmosphere, which are influenced by its temperature, pressure, and moisture content, produce atmospheric refraction. Thus, to understand refraction, we must consider how these meteorological parameters, especially temperature, change with height at various layers above the surface.

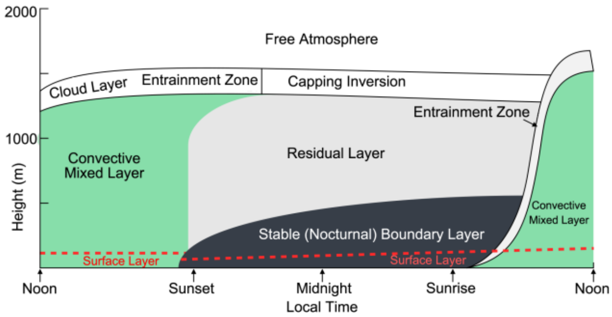

Figure 3 Atmospheric boundary layer. Image by NikNaks from Wikimedia Commons (CC-SA3); after (Stull, 1988).

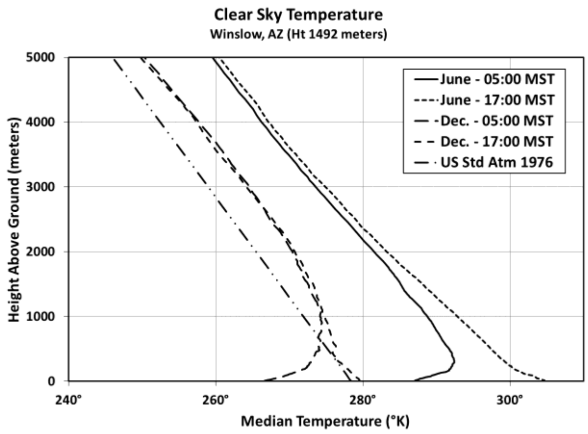

Figure 4 Radiosonde temperature profiles at Winslow, Arizona. Medians of 120 to 138 selected soundings. Data are selected for conditions of clear sky when astronomical observations would be possible, using morning and evening observations near the summer and winter solstices.

The temperature structure near the surface (Figure 3) changes drastically in the course of the day, but during the period from sunset to sunrise, when most archaeoastronomical observations take place, it is characterized by a cold surface layer within 30 to 50 meters of the ground produced by radiative cooling at the earth's surface. Above this is a stable nocturnal boundary layer, extending as high as 500 meters above the ground, in which the temperature continues to increase with height. Above this is a residual layer, where we find the normal cooling with increased height that is produced by daytime convection, and which continues through the night.

Radiosonde measurements from Winslow, Arizona (Figure 4) indicate the general temperature profile at the solstices within a few hours of sunrise and sunset. However, due to instrumental limitations, radiosondes do not adequately record the temperature gradient in the nocturnal surface layer.

Using temperature data recorded at a 60-meter tall instrumented meteorological tower, the temperature gradient near the surface is found (Figure 5) to be extremely strong and variable, especially at the time of sunrise.

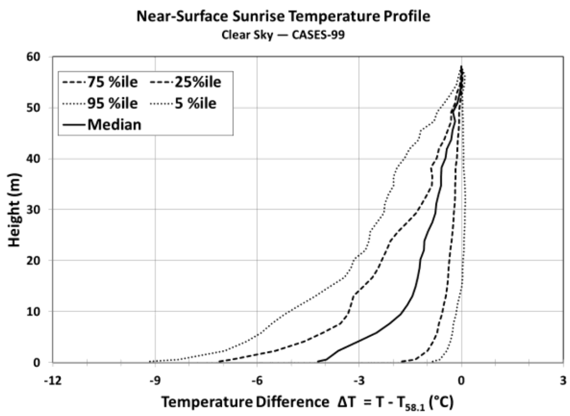

Figure 5 Day to day variability of temperature profiles near the surface at the time of sunrise from the CASES-99 field experiment (Poulos 2002) in southeastern Kansas. Data were recorded every 1.8 meters. Temperatures are presented as differences from the temperature at 58.1 meters above the surface.

Combining temperature data recorded at this tower at the times of sunset and sunrise with local pressure data, we can compute the coefficient of terrestrial refraction (defined as the ratio between the radius of the Earth and the radius of the refracted ray) at sunset and sunrise in the region near the surface (Hirt et al, 2010). We find (Table 1) that the magnitude of refraction near the surface is substantially larger at sunrise than at sunset. This reflects the growing strength and depth of the surface layer in the course of the night.

Table 1 Near-surface refraction at sunset and sunrise. Average value for clear days during the CASES-99 field experiment.

	Magnitude of Refraction			
Height Range	0.23 – 31.1 m		0.23 – 58.1 m	
Time of Day	Sunset	Sunrise	Sunset	Sunrise
Mean Coefficient of Refraction	0.649	1.014	0.647	0.719

Extending this investigation to the day-to-day variability of near-surface refraction, we find (Table 2) that the variability is also markedly greater at sunrise than at sunset, even more than was the magnitude of refraction.

These calculated results, that the magnitude and variability of refraction are greater at sunrise than sunset, agree with monthly measured values, tabulated by Sampson in his master's thesis and a subsequently published paper (Sampson 1994; Sampson et al 2003), in which he measured refraction over nearly flat terrain in the course of a year.

But which of the principal meteorological parameters has the greatest effect on refraction? Drawing on the means and standard deviations of the main meteorological parameters near sunrise and sunset during the month-long CASES-99 study period, we find (Table 2) that a one sigma change of the temperature gradient has a much greater effect on refraction than a one sigma change of the temperature, and even more than a one sigma change of the pressure.

Table 2 Variability of computed coefficient of refraction (Hirt et al, 2010) caused by a one standard deviation change of the principal meteorological parameters near the surface at sunset and sunrise.

Height Range	0.23 – 31.1 m		0.23 – 58.1 m	
Time of Day	Sunset	Sunrise	Sunset	Sunrise
Meteorological Parameter	Variability of Coefficient of Refraction			
ΔT / Δh	±0.312	±0.542	±0.167	±0.345
T	±0.036	±0.039	±0.022	±0.028
P	±0.005	±0.006	±0.003	±0.004

4. TOPOGRAPHIC INFLUENCES

The previous findings for flat terrain are not always applicable. The situation at an elevated ridge or peak is different from the situation in a valley or plain (Figure 6). Atop ridges, where archaeoastronomical horizon markers are commonly located, the flow of cool air from the crest, combined with the greater exposure to winds, reduces the strength and depth of the nocturnal surface layer.

It should be emphasized that although the temperature gradient is stronger and deeper near the surface than on hilltops, the geometric factors, discussed previously, modify the influence of temperature gradients on refraction. Assuming an observer located in a valley and a horizon marker on a hilltop, geometric factors will significantly reduce the effect of the stronger gradient near the observer and enhance the effect of the weaker gradient near the marker; the net effect is to weaken the magnitude of archaeoastronomical refraction.

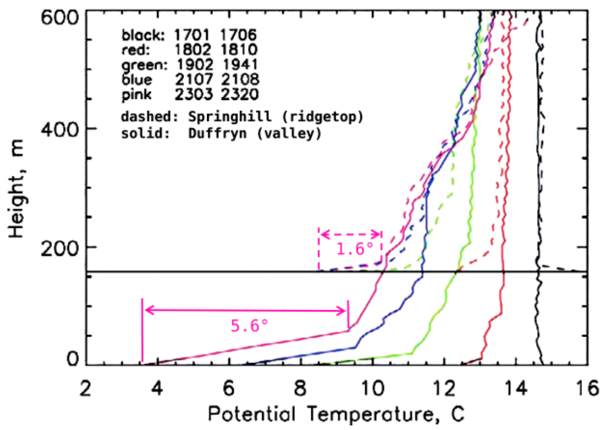

Figure 6 Difference between nearly simultaneous nighttime radiosonde profiles in a valley and atop a ridge. Data from the COLPEX field experiment on the border of England and Wales. Height measured relative to the valley site. Figure based on (Price et al, 2011); used by permission.

Besides modifying the depth and intensity of the meteorological surface layer, the nature of the local topography also strongly influences the length of the portion of the ray path that passes through the surface layer, as can be seen by examination of Figure 1. Further calculations, which will not be discussed here, have demonstrated that local topography can modify the path length near the surface by as much as two orders of magnitude. This difference in path length near the surface produces differences in near-surface refraction greater than two orders of magnitude between elevated markers and flat terrain (McCluskey 2017).

Local topography thus transforms the ways local meteorological conditions interact with the geometry of a site to produce different magnitudes of refraction.

5. CONSEQUENCES

What are the consequences of these findings for archaeoastronomical and ethnoastronomical research?

If we are making low-precision claims for a site, allowing errors of ten arc minutes or more, the details of refraction are small enough to be ignored. A range of standard refraction formulas or tables based on the temperature, pressure, and humidity measured at the observing site, such as those discussed by Fletcher (1952) and Wittmann (1997), are sufficient to correct for refraction.

If we are making high-precision claims for a site, we are concerned, as was mentioned above, with both the consistency and the absolute accuracy of observations possible at the site. For this reason, we need both the magnitude and the variability of archaeoastronomical refraction at the site. As Young (2004; 2006) has pointed out, precise refraction computations at low altitude require knowledge of the actual temperature gradient which as we have shown, depends strongly on the height of the ray above the local topography

To do this, high precision calculations must incorporate an atmospheric model of meteorological conditions at the time of observation as a function of height of the ray above the ground from the observer to the upper limits of the atmosphere. Obviously, we have no records of the weather in the distant past; our model must rely on local and seasonal climatological data of significant meteorological parameters, particularly the temperature gradient, the temperature, the atmospheric pressure, and, if possible, the moisture content. Since we are concerned with the consistency of possible observations, we must consider both the climatological averages and the day to day climatological variation of these parameters (especially of the temperature gradient).

High precision calculations must also incorporate the local geometric configuration of the site, either the angular altitude or the difference in height of the horizon marker, and the distance to the horizon marker. With these it becomes possible to compute the two angles of terrestrial refraction, α and β, and the astronomical refraction ρ, all of which play important roles in determining archaeoastronomical refraction ε.

These high precision calculations must also incorporate the topography along the ray path from the observer to a point where the ray permanently leaves the boundary layer. Our special concern is with the extent of the region or regions in the surface layer where the ray undergoes strong and variable refraction.

For the geometric reasons discussed above, only a small portion of the refraction near the observer contributes to archaeoastronomical refraction and, in many cases is negligible, while refraction near the horizon marker contributes almost exclusively to archaeoastronomical refraction, and so must be considered carefully.

Eventually, it should be possible to employ digital integration to compute both the magnitude and variability of archaeoastronomical refraction at any site. Such a model would allow us to evaluate the suitability of the site for a given class of horizon observations.

Let me close with a very general comment: As is well known, archaeoastronomy is embedded in local contexts. We are used to seeing its ties to local culture and society. However, this investigation of refraction shows how much even these purely physical considerations are dominated by local influences: particularly by local meteorology and local topography.

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